



# Galileo and Changing Views of the Universe

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November 19, 2015

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# Introduction



Most of us are familiar with the fact that Galileo was brought to trial and censored by the church for promoting the view that the earth revolves around the sun. This trial is often viewed as a conflict between Christian faith and science. However, the situation was much more complicated than that.

In 1962 Thomas Kuhn wrote a very influential book entitled **The Structure of Scientific Revolutions**. In this book he describes how historically science has not followed a straight line path of gradual improvement, but instead there have been normal periods where science operates under a structure of shared assumptions and approaches to problems called a paradigm that in due time is interrupted by a revolutionary paradigm shift. A paradigm shift represents a radically different way of thinking about the world. It usually involves more than the rejection of a single idea, but instead it involves changes in multiple interconnected ideas. Initially the new paradigm is vigorously resisted while support for the new paradigm is being developed. Eventually there follows a new normal period where the new paradigm prevails.

Galileo was part of a major paradigm shift. Prior to the 16th century, the scientific view of the universe was based primarily on the ideas of Aristotle (384–322 B.C.) and the later refinements of these ideas by Ptolemy (c.85–165 A.D.). They assumed among other things that the earth was stationary and was the center of the universe. The sun, moon, and the other planets were assumed to revolve around the earth. They also believed that the moon and everything beyond were part of a perfect, unchanging region where all the stars and planets were composed of a special weightless material called aether, unlike anything found on earth.

It is remarkable that this way of viewing the universe was commonly accepted for almost 2000

years. Although we now know that most of the ideas of Aristotle and Ptolemy about the universe we live in were wrong, there were very powerful common-sense arguments for believing them at the time. The first real challenge to the Aristotelian view was presented by Copernicus. Nicolas Copernicus (1473–1543) developed a model for the solar system in which the earth and the other planets orbited around the sun. Although the model he developed was as complicated as that of Ptolemy and was no more accurate, it did seem to provide better explanations of certain features. This stimulated other scientists to think about better ways of describing the heavens.

Galileo was a supporter of the Copernican view and offered several pieces of evidence against the Aristotelian viewpoint. Using the recently invented telescope he observed such things as mountains on the moon, several moons orbiting Jupiter, sun spots, and phases of Venus. Although these observations were not sufficient to completely overthrow the current paradigm, they did present significant problems for the Aristotelian viewpoint.

However, there still remained a number of common-sense objections to a sun-centered system such as

- Why don't we feel like we are moving?
- Why don't we feel a wind as we move through the air?
- Why don't objects fly off a rotating earth?
- What is powerful enough to move the earth and keep it moving?
- Why don't we observe stellar parallax if we orbit the sun?

These objections were eventually answered, but it took a new formulation of physics by Newton (1643–1727) and more accurate measurements to provide the answers.

As you can imagine, Galileo's advocacy of the Copernican model met stiff resistance both from the academic community and the church. If anything, the resistance from the academic community was the stronger of the two. Galileo had a number of backers within the church community, but very few in the academic community. The resistance from the church stemmed from the fact that Thomas Aquinas (1225–1274) had incorporated many of Aristotle's views into a Christian theology that became part of official church doctrine. This theology was consistent with several passages in the Bible that seemed to support an earth-centered view.

The church didn't object to a sun-centered model of the universe as long as it was treated as a computational tool and was not presented as representing reality. On the other hand, members of the academic community saw this paradigm shift as a threat to the cherished beliefs on which their careers were founded.

In this paper we will look at the reasons behind the long tenure of the Aristotelian viewpoint as well as the work of Copernicus, Galileo, Kepler, Brahe, and Newton that eventually lead to its downfall. We will also look at the trial of Galileo and some of the events that led up to it. Finally, we will see how the Christian church eventually modified its interpretation of certain passages of

scripture that seemed to support a stationary earth. It was realized that the language used in these passages didn't have to be taken literally, but could be interpreted in a poetic or phenomenological manner. The paradigm shift brought about changes in both science and Christian theology. In both cases the changes were resisted initially but eventually converged toward a peaceful solution.

# **Ancient Astronomy**

In this chapter we will look at how ancient cultures viewed the universe. In particular we will examine the views of Aristotle and Ptolemy that were widely accepted for almost 2000 years.

# Movement of the Sun, Moon, Stars, and Planets



From early times man has had an interest in the objects he saw in the heavens. Great significance was often attached to the ways these objects seemed to move. In this section we will look at how the sun, moon, stars, and planets appear to move as viewed from the earth. Any successful model of the universe must accurately predict these movements.

**Movement of the Stars** Each of the stars follows a circular path across the sky that repeats about every 24 hours. However, the stars move together so that their relative positions remain unchanged. For this reason they are often referred to as the fixed stars. The star Polaris appears to be fixed to viewers in the northern hemisphere and there are polar stars that appear to be fixed to viewers in the Southern hemisphere. Thus, the movement of the stars is exactly what would be expected if all the stars were located on a giant rotating sphere whose axis passes through Polaris and the southern polar stars and which makes a complete rotation every 24 hours. The fixed stars do not appear to change in brightness.

**Movement of the Sun** The Sun appears to follow a circular path each day, rising in the east and setting in the west. However, the sun's position relative to the fixed stars changes slightly each day (about  $1^\circ$  per day). This daily movement is in an eastward direction relative to the fixed stars and the sun completes a cycle through the stars in a year. The path of the sun through the fixed stars lies in a plane that is inclined about  $23.5^\circ$  to the plane of the equator. The sun passes through different constellations at different times of the year, and this is what gave rise to the signs of the zodiac. Of course, the position of the sun relative to the fixed stars can only be observed for a short time before sunrise and after sunset.

**Movement of the Moon** The moon, like the sun, moves daily relative to the fixed stars, but its motion is faster. Its path is inclined about  $5^\circ$  to that of the sun and it completes a cycle in about 27.3 days. However, since the sun has moved during the moon's cycle, it takes a short time for the moon to realign itself with the sun. Thus, the time between full moons is about 29.5 days. The phases of the moon are due to the fact that different portions of the illuminated half of the moon are visible to an observer on the earth when the moon is in different parts of its orbit.

**Movement of the Planets** There were five planets (not counting the sun and moon) that were known to ancient astronomers—Mercury, Venus, Mars, Jupiter, and Saturn. On any given night a planet (when it is visible) moves just like one of the fixed stars. You may have heard that stars flicker and planets don't. There is some truth in this, but it is difficult to identify planets on this basis alone. The main thing that distinguishes planets from stars is that on successive nights they, like the sun and moon, move slowly eastward relative to the fixed stars. The paths of the planets lie within  $8^\circ$  of the path of the sun; so they pass through the same constellations.

Each of the planets at certain times exhibits retrograde motion in which it stops its eastern migration and moves for a while in the reverse direction before stopping again and then resuming its eastward migration. The planets Mercury and Venus never appear very far from the sun. Mercury is always within  $28^\circ$  of the sun and Venus is always within  $46^\circ$  of the sun. As the sun makes its yearly journey, Mercury and Venus move back and forth across the sun.

Unlike the stars, the planets do change brightness. They are brightest during their retrograde motion. In addition the movement of the planets seems to be faster at certain times than at others. This complicated movement of the planets is what makes modeling their movement versus time difficult.

## Aristotle(384–322 B.C.)



In this section we will look at Aristotle, whose views on the universe were dominant for almost 2000 years. Aristotle was born in a small Greek colony in northern Greece called Stagira. His father was the personal physician to Philip of Macedon, the grandfather of Alexander the Great. Presumably, it was his father who taught him to take an interest in the details of natural life. At the age of eighteen, Aristotle became a student at the Academy of Plato. After Plato's death, Aristotle spent four years as a tutor of Alexander (later to become Alexander the Great). Later on, during his military conquests, Alexander helped to spread Greek culture (including Aristotle's work) to other regions. In 335 B.C. Aristotle returned to Athens and started his own school called the Lyceum.

Aristotle was interested in almost everything—government, ethics, philosophy, science, etc. He pretty much invented modern logic. Although Aristotle had great admiration for Plato, he disagreed with his teacher on a number of things. One of these disagreements was on the nature of forms (ideals).

Plato was searching for a certain and unchanging basis on which to base knowledge. Thus, he postulated an unchanging world of ideals or forms for which visible objects were merely an imperfect representation. For example, there is an ideal concept of a chair that is universal and applies to all chairs at all times. However, there are many different representations of a chair in the visible world. Central to the concept of forms was the idea of purpose. For example, the purpose of a chair was to provide a place to sit. To Plato these forms could only be approached through reason.

Aristotle also believed in universals and ideals, but he didn't believe that they could be separated from the visible world. In fact, he believed that these ideals or forms could only be recognized by studying the real world of concrete objects. Aristotle used the word 'essence' for these ideals, and believed that every substance was a unity of essence and matter.

**Aristotle's Science** Aristotle's primary scientific interest was in biology. He observed and cataloged a large number of plants and animals, and divided them into classes. He observed how living things developed. For example, an acorn always developed into an oak tree. Thus, he believed that the acorn had built into it an essence or purpose that was fully realized when it became an oak tree. He transferred this idea of essence to inanimate objects as well.

He thought that all objects here on earth were made up of four basic elements—earth, water, air, and fire. He believed that the essence or goal of the earth element was to move toward the center of

the universe which he took as the center of the earth. That is why heavy objects containing a lot of the earth element tend to fall toward the ground. The essence of water is also to move toward the center of the universe, but this nature is not as strong as it is for earth. Air and fire have a natural goal to rise above the earth. This natural essence is greater for fire than it is for air.

There was one class of objects that didn't fit into this scheme, namely the stars and planets. They seemed always to be moving in a circular path at a uniform speed. Therefore, Aristotle divided the universe into two regions: the region consisting of the earth and everything in between the earth and the moon (sublunar region) and the region beyond the moon (the superlunar region). The sublunar region is composed of the four basic elements. The superlunar region is composed of a fifth weightless element called aether or quintessence. The nature of aether is to move in a circular path with a constant speed. It seems strange to us to assign purposes and goals to inanimate objects, but to Aristotle this was what caused objects to behave as they do.

Instead of avoiding motion and change as Plato did with his world of unchangeable forms, Aristotle embraced change and sought out causes for change and motion. Aristotle considered two types of causes for motion. There is natural motion that is due to an object's built in essence and there is motion that is caused by an outside agent. He called motion produced by an external agent 'violent motion.' All motion in the sublunar region eventually stops. Violent motion stops when the external agent stops pushing or pulling. Natural motion stops when the object has reached its natural place or when it is prevented from doing so by another object, e.g. a falling rock stops when it hits the ground.

**Aristotle's View of the Universe** Based largely on Aristotle's ideas a view of the physical universe was developed that was the dominant view from about 300 BC up into the sixteenth century. Here are some of the components of this view

1. The universe is eternal. It had no beginning and no foreseeable end.
2. The earth is spherical.
3. The earth is located at the center of the universe.
4. The earth is stationary, i.e., it doesn't rotate or revolve around any other object.
5. The stars, the moon, the planets, and the sun revolve around the Earth, completing a revolution about every 24 hours.
6. The sublunar region is composed of four basic elements: earth, water, air, and fire. This region is subject to change and decay.
7. The superlunar region is composed of a fifth element called aether whose natural motion is circular at a uniform rate. The superlunar region is perfect and unchangeable.
8. All motion in the sublunar region eventually stops.
9. An object that is stationary will remain stationary unless there is some source of motion.

**Spherical Earth** You may be surprised that it was realized at this early date that the earth is spherical. However, there were several observations that led the ancient Greeks to this conclusion.

- The sun rises and sets at different times as you move in an east-west direction.
- The positions of constellations in the sky change as you move in a north-south direction. New constellations may come into view and others may disappear.
- As a ship approaches land the land appears to rise out of the sea. High points appear above the horizon first and later on the lower regions.
- During a lunar eclipse, the shadow of the earth on the moon appears circular.

In fact Eratosthenes measured the circumference of the earth in about 240 B.C. He knew that the sun would be directly above the city of Syene at noon on the summer solstice. He was sure that the sun was directly overhead when he could see the reflection of the sun at the bottom of a deep well. He placed a vertical pole in the city of Alexandria which was 5000 stadia (about 925 Km or 270 miles) due north of Syene.

By looking at the shadow cast by the pole he determined the sun's rays made an angle of  $7.2^\circ$  with the pole. It can be seen in Figure 1 that  $7.2^\circ$  is also the angular separation of the two cities. As the ratio  $360^\circ/7.2^\circ=50$ , it follows that the ratio of the circumference to the distance between the cities is also 50. Thus the circumference is  $50 \times 5000$  stadia or 46,250 Km. Current Satellite measurements give a value of 40,008 Km for the circumference. Thus, he was off by only about 16%.

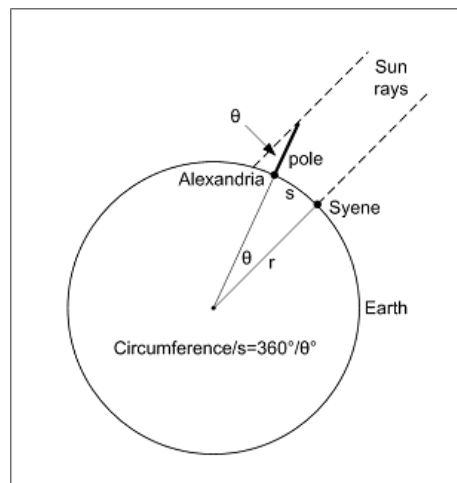


Figure 1: Eratosthenes measures the earth's circumference.

This experiment by Eratosthenes is further verification of a spherical earth as the pole would have cast no shadow if the earth were flat. The Greek's idea of a spherical earth spread to other regions, and by 800 A.D. it was rare to find any people group that believed in a flat earth.

**Stationary Earth** There are a number of observations that led to the belief that the earth is stationary.

- a. It doesn't feel like we are moving.
- b. We don't feel the wind-like effects of the air as we move through it.
- c. Dropped objects fall straight down. It was believed that if we were moving, then we would have moved ahead a certain distance as the object was falling and the object would land behind us.
- d. The earth is very big and heavy, and there is no obvious source capable of moving it.
- e. We don't observe stellar parallax (the change of relative positions of stars due to changes in the observer's position).

Although the reasons given for a stationary earth seemed very reasonable at the time, most of us today believe that the earth does move and that it revolves around the sun. However, you might ask yourself why you believe this. For most of us, it is because someone told us or because we read it in a book. It may surprise you that even today we don't have any direct observations that we revolve around the sun. As we will see, the reasons that this is the accepted viewpoint today are very subtle and not at all obvious.

**It Doesn't Feel Like We Are Moving** It can be calculated that the earth is revolving around the sun at about 70,000 mph and is rotating about its axis at about 1000 mph at the equator. Yet we don't feel like we are moving. The reason we don't feel the motion is that the earth, ourselves, and the atmosphere are moving together at a fairly constant speed. If we are traveling in a vehicle at a constant speed and there is very little vibration, we don't feel like we are moving. What we feel is change of motion (acceleration).

The fact that we don't feel a strong wind is due to the fact that the atmosphere is by-and-large moving with the earth. There have been various reasons proposed for why the atmosphere moves with the earth. Gravity keeps the atmosphere from escaping from the earth and would tend to keep it moving along the earth's orbit around the sun. However, gravity doesn't force the atmosphere to rotate with the earth. This is thought to be due primarily to viscosity. Since the earth's surface is very uneven, the atmosphere near the surface is dragged along as the earth rotates. The viscosity of the atmospheric gases (even though small) eventually causes the rest of the atmosphere to follow along. There are of course winds that are due primarily to temperature variations. However these winds are very small compared to the winds that would be present if the atmosphere didn't move with the earth.

**Earth at Center** The idea of the earth being the center is perfectly natural if the earth is stationary since all the heavenly bodies appear to revolve around the earth. Belief in the earth as the center was not motivated primarily by the belief that the center was a place of great importance, but

was believed primarily on physical grounds. In fact, the superlunar region was considered to be the place of greatest perfection and importance. The earth was characterized by imperfection and decay.

**Stellar Parallax** The idea of parallax probably requires some explanation. If you hold up a piece of paper in front of you and hold up a finger in front of the paper, you will notice that your finger appears to move from one side of the paper to the other as you move your head from side to side. This phenomenon is called parallax and involves the apparent change of the relative positions of objects due to changes in position of the observer. If the earth were circling the sun, the relative position of stars should exhibit parallax when viewed from different positions in the orbit. Since none was observed, this was taken as evidence of a stationary earth. It turns out that parallax is real, but because the stars are so far away our instruments were not accurate enough to measure it until the year 1838 A.D. Stellar parallax is probably the best empirical evidence we have today that the earth revolves around the sun.

**Falling Objects** The reason that objects appear to drop straight down is that the velocity of the dropped object has two components—the downward velocity produced by gravity and the velocity in the direction of motion that it shares with us. The velocity it initially shares with us persists as the object falls, since there is no force to change it (the principal of inertia). Thus, the dropped object not only moves downward, but also moves forward just as we do. A true understanding of relative motion and inertia was not obtained until the 16th and 17th centuries (Galileo and Newton).

**Earth Large and Heavy** Everyone realizes that it takes considerable effort to move a heavy object such as a large rock. The earth was certainly a very large and heavy object, and there didn't appear to be any agent around powerful enough to move it and to keep it moving. The concept of inertia was not understood until the time of Galileo and Newton. Newton's laws imply that once an object is in motion it takes a force sufficient to overcome its inertia in order to bring it to a stop. The earth was set in motion during the creation of the solar system.

The major force acting on the earth today is the gravitational attraction of the sun. This force causes the earth to speed up in certain parts of its orbit and to slow down in others, but it never acts in such a manner as to bring it to a halt. In fact it can be shown that the earth-sun system is periodic. As an aside, the idea of gravitational attraction itself is somewhat mysterious. How can an object exert a force on another object when there is no contact? Many thought that Newton was engaging in witchcraft when he first proposed the idea.

Although we now believe that most of Aristotle's beliefs concerning the solar system are false, they seemed to be very reasonable at the time. In fact these views were not seriously challenged until the 16<sup>th</sup> century.

**Composition of Stars and Planets** Today we know that stars consist of hot gases and planets are composed of hard materials much like we have on earth. However, prior to the development

of the telescope there was no reason to think that the lights seen in the sky were made of materials like those on the earth. The fact that the stars moved in a regular pattern year after year led to the idea that the superlunar region represented perfection. The motion of the stars was very different from observed motion on the earth which always eventually comes to a halt. Thus, these ideas of Aristotle seemed reasonable prior to the time of Galileo.

## Ptolemy (c.85–165 A.D.)



Aristotle's view of the solar system was greatly expanded by Claudius Ptolemy. Not much is known about Ptolemy's life except that he was a mathematician and astronomer that lived in Alexandria, Egypt during the last part of the first century A.D. into the second century A.D. At that time Egypt was a Roman province. His famous work was the *Almagest* in which he laid out a model for the motion of the planets that was based on the ideas of Aristotle. Each planet was modeled separately. To handle the retrograde movement of the planets he introduced epicycles. An epicycle is a smaller circle whose center moves in a circular orbit around the earth.

Like Aristotle, Ptolemy took the earth as the center of the universe. As such, the earth was the center of the sphere containing the fixed stars. However, the earth was not taken as the center of a planet's orbit. A planet moved at a constant angular velocity around an epicycle whose center moved at a constant angular velocity around a larger circle whose center was displaced somewhat from the earth. Epicycles were introduced primarily to handle retrograde motion.

It was observed that planets moved faster in some parts of their orbit than in others. To handle this nonuniform motion, Ptolemy introduced another point called the 'equant.' It was assumed that the angular velocity relative to this point was constant. This produced a nonuniform angular velocity relative to the center. Figure 2 shows the basic parts of Ptolemy's model of planetary motion.

Since Mercury and Venus always appeared close to the sun, Ptolemy assumed that the centers of their epicycles lay on the line connecting the earth and the sun and that they rotated around the earth at the same rate as the sun does. This is illustrated in Figure 3.

The retrograde motion of the outer planets (Mars, Jupiter, and Saturn) always occurs in the night sky and the planet is always brightest during this retrograde motion. To accomplish this Ptolemy required that the arrow from the center of a planet's epicycle to the planet always be parallel to the arrow from the earth to the sun. This configuration is shown in Figure 4.

Using models such as this, Ptolemy was able to predict the motion of the planets fairly accurately. Sometimes he needed to introduce epicycles riding on other epicycles to get the desired accuracy. Ptolemy's planetary model was the generally accepted model for planetary motion up into the 16<sup>th</sup> century. It is not known how Ptolemy viewed his planetary model. Did he think that his model represented reality, or was it merely a computational tool for producing results that agree with observation. Even today Scientists often disagree on whether certain theories really represent reality or not. At the time of Galileo the church didn't object to a sun-centered universe as a

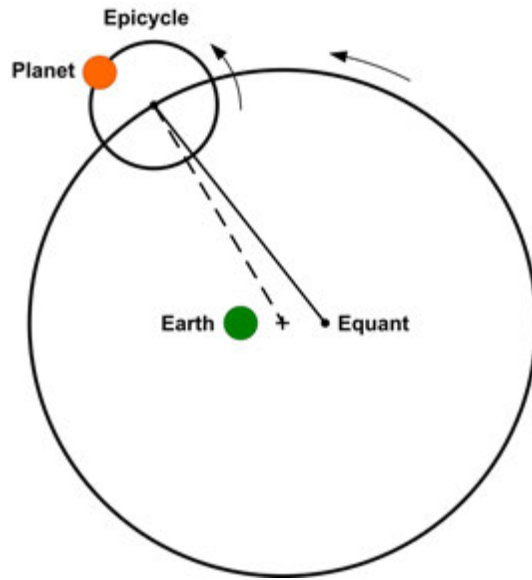


Figure 2: The planetary model of Ptolemy

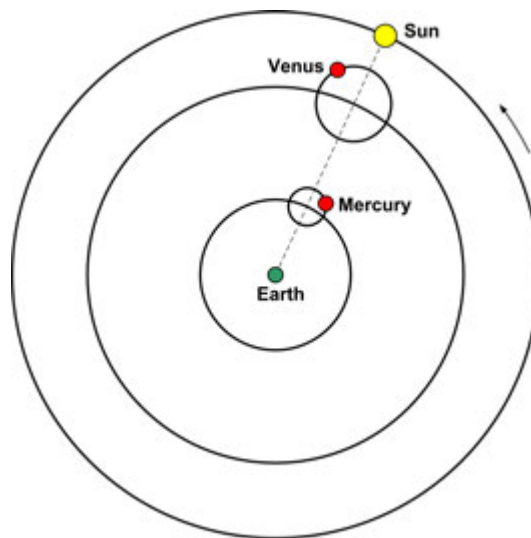


Figure 3: Mercury and Venus according to Ptolemy's model.

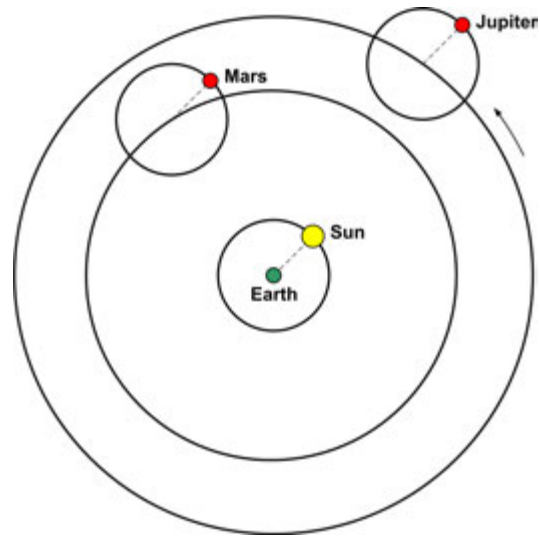


Figure 4: Outer Planets according to Ptolemy's model.

computational tool, but it did object to claims that it represented reality.

There were two aspects of Ptolemy's models that persisted in the sun-centered models of Copernicus. These were the requirements that all motions be circular and at a uniform speed. These requirements were not dropped until Kepler developed his laws of planetary motion.

# Problems Emerge

In this chapter we will see how the views of Copernicus (1473–1543), Galileo (1564–1642), and Tycho Brahe (1546–1601) challenged the prevailing Aristotelian viewpoint. We will also look at the trial of Galileo and the events leading up to it.

## Copernicus (1473–1543)



Nicolas Copernicus was a Polish mathematician and astronomer who served as Canon (an administrative position) for the church in Frauenberg. In the early 1500s he developed a sun-centered model that was the first real challenge to the long accepted model of Aristotle and Ptolemy. His major work was called **Revolutions of the Celestial Spheres** which was published just before his death in 1543. A Lutheran Minister Andrew Osiander handled the publication and inserted a preface without Copernicus' knowledge. In this preface he wrote that Copernicus was merely offering a hypothesis, not a true account of the workings of the heavens. Although this clearly did not represent Copernicus' viewpoint, it probably protected this work from close scrutiny by the church.

Copernicus' model offered the advantages that it didn't need epicycles in order to produce retrograde movement and it didn't need an equant point in order to produce non-uniform motion. However, epicycles were added in order to obtain accurate predictions. The accuracy of his model was good, but it was no better than Ptolemy's model. As far as complexity, Copernicus' model (including epicycles) was at least as complex as Ptolemy's model. Figure 5 shows Copernicus' model for Mars. The models for Jupiter and Saturn were similar to that of Mars, but the models for Venus and Mercury were even more complex.

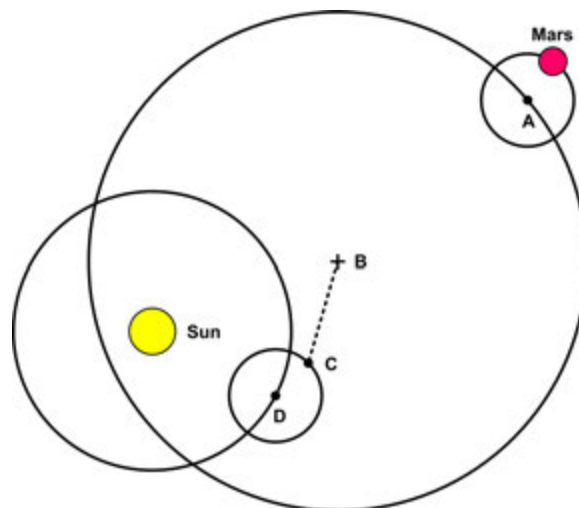


Figure 5: Copernican model of Mars

In this model Mars follows a circular path around the point A. The point A follows a circular path around the point B. The point B also moves, but always stays in the same position relative to the

point C. The point C is the center of the circular path followed by the Earth (not shown). C follows a circular path around the point D which in turn follows a circular path around the Sun.

As with Ptolemy's model, Copernicus assumed that the points moved around their circular paths at a constant speed. He also assumed that the earth rotates about its axis producing the daily changes.

The Copernicus model had a simpler explanation for retrograde motion. Let us take Mars as an example. Mar's orbit is farther from the sun than Earth's orbit and it takes about two years for a complete rotation. Thus, the Earth moves faster and passes Mars about every two years. It is not hard to verify that retrograde motion of mars as seen from the earth happens during this passing. In addition mars will seem brighter during this retrograde motion since they are closest at this time.

All of the ancient models were committed to circular motion at constant velocity. Ptolemy was only able to obtain constant velocity by introducing an equant point. Copernicus considered that his greatest contribution was the elimination of the equant. His model also had a natural explanation for why Mercury and Venus always appeared near the sun. In the Copernican model the orbits of Mercury and Venus were closer to the sun than the Earth's orbit.

In the years that followed, a number of astronomers used Copernicus' model to make predictions. However, most viewed his model as a computational tool and not a representation of reality. The common sense arguments of Aristotle still seemed to support a fixed Earth. As we will see in the next section, it was Galileo who began to challenge many of the long held Aristotelian assumptions and revived interest in the Copernicus model.

## Galileo Galilei (1564–1642)



Galileo's observations with a telescope challenged several of the Aristotelian assumptions. Prior to this time all of the observations of the stars and planets were made with the naked eye.

Galileo was born on 15 February, 1564 in Pisa which was part of the Duchy of Tuscany. He was the first of six children. His father Vincenzo was a music teacher and an accomplished player of the lute (a stringed instrument). Galileo also became a very good lute player. In his teen years Galileo was sent to a monastery for his education. Galileo was attracted to the contemplative life style there and became a novice, intending to join the order. His father didn't approve and withdrew his son. In 1581 Galileo was enrolled at the University of Pisa to study medicine as his father desired.

In 1582–1583 Ostilio Ricci, the mathematician of the Tuscan court, taught a course at the university on Euclid's elements which Galileo attended. Galileo began studying the elements on his own and asked many penetrating questions. Ricci attempted to persuade Vincenzo that his son should pursue a career in mathematics. Vincenzo was not convinced, but did allow his son to be tutored in mathematics while continuing his education in medicine. This didn't work out well and in 1585 Galileo dropped out of college without completing his degree.

For the next few years Galileo taught mathematics privately in Florence and later in Siena. In 1587 Galileo traveled to Rome to meet with Clavius who was professor of mathematics at the Jesuit Collegio Romano there. He showed the professor some methods he had developed for calculating the center of gravity of various solids and Clavius was very impressed. The two remained in correspondence after Galileo left Rome. Galileo also corresponded regularly with another mathematician Guidobaldo del Monte. Through the influence of Clavius and Guidobaldo, Galileo was appointed to the chair of mathematics at the University of Pisa in 1589. He held this position for three years. While at the university he wrote *De Motu* a series of essays on motion which he never published. One important idea contained in *De Motu* was the use of inclined planes to test theories of falling bodies at slower speeds.

In 1591 Galileo's father Vincenzo died and Galileo had to assume the financial support of the family. His salary at the university was not sufficient, so he sought another place of employment. Again through the influence of Clavius and Guidobaldo he was appointed professor of mathematics at the University of Padua at three times the salary. At Padua his duties were mainly to teach Euclidean geometry and standard (geocentric) astronomy to medical students. He taught there for eighteen years.

In 1604 he gave three public lectures concerning the appearance of a new star (now known as Kepler's Supernova). Using a parallax argument he showed that the new star could not be in the lunar region close to the earth. This contradicted the Aristotelian assertion that all changes in the heavens must occur in the region between the earth and the moon. Galileo had also questioned other Aristotelian assumptions. In particular, he didn't believe that the speed of falling objects was proportional to their weight. As a young boy he had observed that hailstones of different sizes appeared to fall at the same rate. It is not sure when Galileo became a believer in the theory of Copernicus, but in a 1598 letter to Kepler he stated that he was a Copernican. However, he didn't state that publicly until much later.

A working telescope was first unveiled in the Netherlands in 1608. Initially it was used for military applications. In 1609 Galileo began using the telescope for astronomical observations. He made a number of refinements of the instrument and eventually achieved a magnification of 32x. His observations with the telescope challenged many of Aristotle's assumptions. His observations of the moon revealed an uneven (mountainous) surface much as we have on the earth. Figure 6 shows drawings he made of the moon surface from his observations.

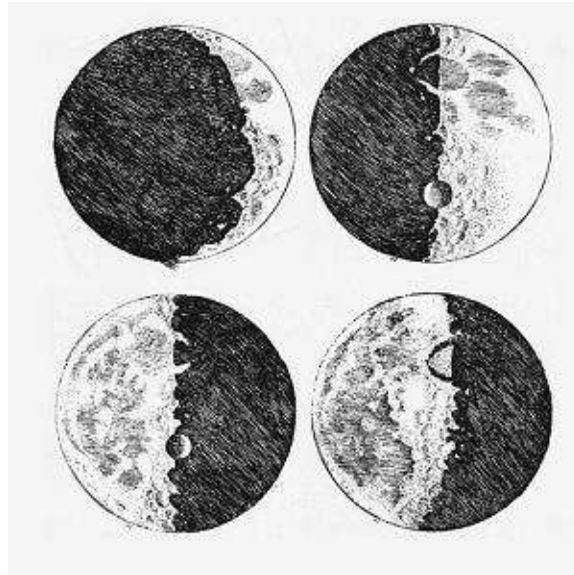


Figure 6: Galileo Moon Drawings

Galileo's observations of the moon also showed that you could have a large Earth-like body that was in motion. This challenged some of Aristotle's assumptions for a fixed Earth.

He also observed objects he correctly identified as moons revolving around the planet Jupiter. This challenged the view that everything revolved around a stationary earth. Here was an example of objects revolving around another object that was itself moving. He also observed that the Milky Way was made up of many tiny stars. In 1610 he published a booklet called the *Starry Messenger* (Sidereus Nuncius) describing his initial telescopic observations. Shortly afterwards he was appointed the Chief Mathematician and Philosopher to the Grand Duke of Tuscany.

Galileo was also the first to observe sun spots through a telescope. This challenged the assumed

perfection of the superlunar region. He also observed that the planet Venus went through a complete set of phases like those of the moon. This was consistent with the sun-centered model of Copernicus (1473–1543), but was not consistent with Ptolemy’s model. In Ptolemy’s model Mercury and Venus had orbits between the earth and the sun. Since they always appear close to the sun, the centers of their epicycles must revolve around the earth at the same rate as the sun does. Thus, Ptolemy’s model would never show a wide range of phases. There is always a significant portion of the dark side of Venus facing the earth. You can see this from Figure 7.

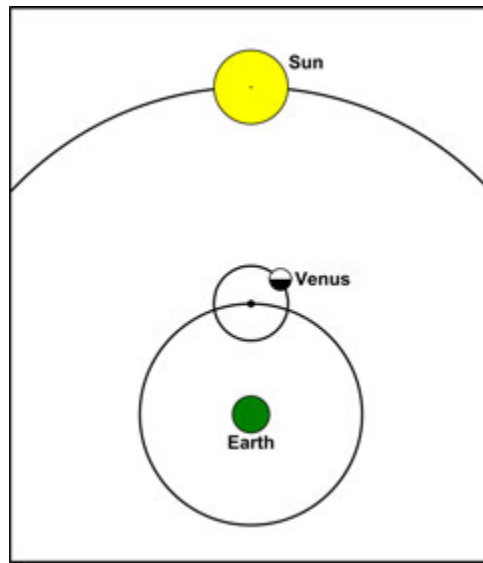


Figure 7: Venus phases in Ptolemy’s model.

Figure 8 shows the phases of Venus in the Copernican model (without epicycles). Here an observer on the earth will see a wide range of phases. Galileo’s observations were repeated by Jesuit astronomers, and they confirmed what he saw.

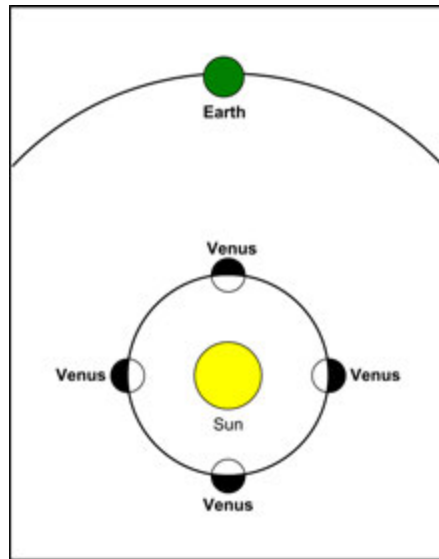


Figure 8: Venus phases in Copernican model

Galileo viewed his observations as a confirmation of the sun-centered Copernican model. Although his observations did pose a serious threat to the model of Aristotle and Ptolemy, there was another earth-centered model that produced accurate results and was also consistent with Galileo's observations. This was the model proposed by the Danish astronomer Tycho Brahe.

## Tycho Brahe (1546–1601)



Tycho Brahe was a Danish astronomer who was well known for his accurate measurements of the heavens. He saw the benefits of both the Copernicus model and Ptolemy's model. He proposed a model that was a combination of the two. He assumed that the earth was stationary and was located at the center of the universe. The sun and the moon revolved around the earth. The other five planets revolved around the sun. Figure 9 shows his model (without epicycles).

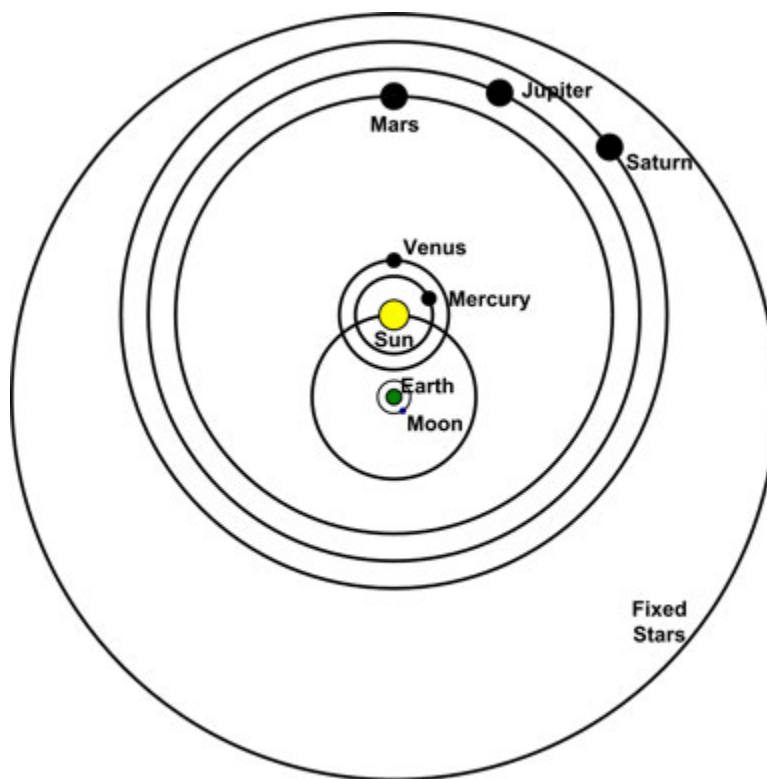


Figure 9: Planetary model of Tycho Brahe

It is not obvious, but it can be shown that Tycho Brahe's model is mathematically equivalent to the Copernicus model. Therefore, they have the same accuracy. The model of Tycho Brahe had the advantage of being consistent with Ptolemy's intuitive arguments for a stationary earth. Thus, at this point in history, belief in an earth-centered universe was still a very rational position. Let us look briefly at the man behind this model.

Tycho was born on December 14, 1546 to Otte and his wife Beate in the family manor, Knutstorp Castle (then in Denmark but now part of Sweden). A picture of this manor is shown in Figure 10. His birth name was Tyge, but he would later go by the latinized name Tycho (pronounced Tee-ko).



Figure 10: Birthplace of Tycho Brahe

Tycho's parents were part of the Danish aristocracy. Tycho was the second born of twelve children. He was very close with his younger sister Sophia who later assisted him in some of his experiments. In 1549, when Tycho was two years old, he was kidnapped from his home by his paternal uncle Jorgen. It turns out that Otte had once drunkenly promised his childless brother one of his children. Jorgen felt that it was now time to collect. Although Otte was upset, he never sought the return of Tycho. Tycho's new parents were also members of the Danish aristocracy. Thus Tycho never had to worry about finances.

When Tycho was 12 years old he was enrolled in the University of Copenhagen (This was not unusual at the time). Following his parents' wishes he majored in law and philosophy. In August of 1560 he witnessed a solar eclipse that had been accurately predicted by the astronomers. He was fascinated that they could accurately predict an event such as this in advance. This was probably the beginning of his passion for astronomy.

Unknown to his parents, Tycho lead sort of a double life while in college. In the daytime he concentrated on subjects related to the major chosen by his parents. In the evening he spent his time reading books on astronomy and making observations of his own. His deception was aided by some of his professors who provided him with books and observation devices. When his parents found out, they moved him to the University of Leipzig in Germany. For the next three years, his curriculum was restricted to courses in law and philosophy, as well as some standard science and mathematics. Brahe's parents employed a 20-year-old Dane to keep watch over him. However, Tycho still managed to secretly continue his study of astronomy. At age 16 he built a device called a Jacobs-staff to measure the angle between stars. This device is pictured in Figure 11.

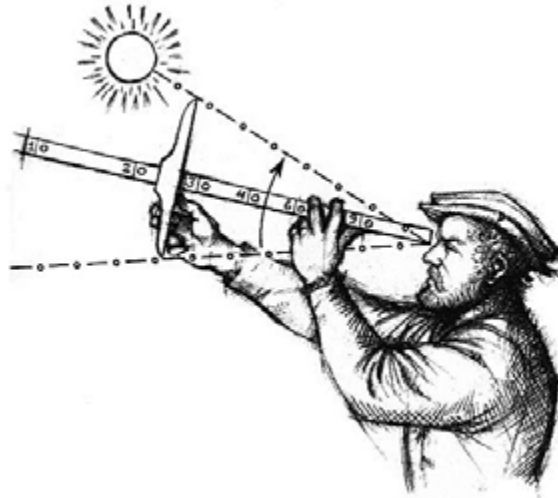


Figure 11: Jacobs-staff measuring device.

It consists of a long piece that is marked like a ruler. The cross piece slides along the long piece. It is moved so that when viewed from the end of the long piece it just fills the gap between the two stars. The angle can be determine knowing the length of the cross piece and the position of the cross piece along the long piece. He also used a standard drafting compass to measure angles. He positioned his eye behind the center of the compass and pointed the legs of the compass to the two different stars.

In August of 1563 Tycho began to keep a log of his observations, He soon began to see errors in many of the published star charts. He came to believe that there was a need for a series of measurements from the same location over a period of years. One of the most glaring discrepancies involved the prediction of the conjunction of Jupiter and Saturn that occurs about every 20 years. The predictions by various astronomers differed widely with some off by months. In September he actually witnessed this conjunction which occurred far ahead of predictions. This motivated Tycho to continue with his observations.

In May of 1565 Tycho took a break and returned home. About a month after his return his father died. Jorgen had been riding with king Frederick II when the king's horse became spooked and threw the king off into the river. Jorgen dived in to the icy waters and rescued the king. Unfortunately, Jorgen contracted pneumonia that lead to his death. The king was very grateful and would later offer assistance to Tycho. Since Tycho was several months shy of his eighteenth birthday, he was considered a minor and his custody was returned to his birth parents Otte and Beate.

Although Tycho's birth parents desired for him to pursue a career in line with their position, they did not make it mandatory. Tycho ignored their wishes and pursued his studies and observations in astronomy at several universities. While at Rostock He and a fellow student had too much to drink and after a disagreement decided to settle it in a duel. In the duel the other student accidentally cut off most of Tycho's nose. Tycho later shaped a piece of brass to replace his nose. He painted it to match his skin color and held it place with paste. Hereafter he always carried paste so he could make repairs. Tycho spent the next several years furthering his education in astronomy and

conversing with other astronomers on how to make better observations.

Near the end of 1570 Tycho went home to be with his father who was dying. Otte died on May 9, 1571. Tycho inherited a large sum of money as well as his birth home Knutstorp Castle. Tycho decided to not live there, but instead went to live with his uncle Steen Bille in the former Monastery of Heridsvad. His uncle along with his sister Sophia were the only family members who supported his ventures into astronomy. To show support for his nephew uncle Bille financed the construction of an observatory on the monastery grounds.

In the early months of 1572 Tycho became romantically involved with a childhood friend Kirsten Jorgensdatter. They wanted to marry, but that would require Tycho to give up all his noble privileges since Kirsten belonged to a lower class. Instead they lived together unwed for three years at which point they were considered married by common law.

In November of 1572 Tycho discovered a brilliant star in the constellation Cassiopeia that was not there previously. He tracked the star for months using a new sextant he had built. He determined that the star must be beyond the moon since it behaved like the fixed stars. This new star is now called “Brahe’s Supernova.” In 1573 he published a book on this new phenomena entitled *De nova stella*.

Figure 12 below shows a sextant like Tycho used at his observatory to measure the angular elevation of a star or planet. The sighting bar can be rotated along a  $60^\circ$  arc. The top of the arc and the end points of the bottom bar are vertices of an equilateral triangle. Thus, the bottom bar is level if a plumb line from the top of the arc bisects the bottom bar.

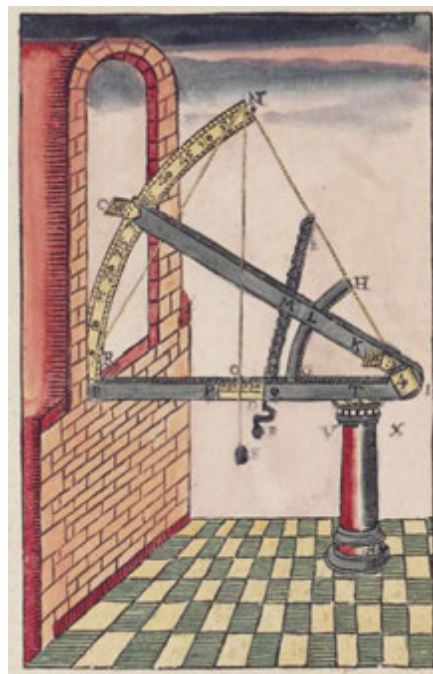


Figure 12: Illustration of a Sextant

In 1574 Tycho was appointed Official Astronomer of Denmark, a title he held for over two decades.

King Frederick II also gave Tycho the island of Hven and provided funds for a large mansion and observatory on the island. Tycho constructed a large castle on the island which he called Uraniborg. He also constructed a fully equipped observatory nearby. Figure 13 shows Tycho's Uraniborg castle.



Figure 13: Uraniborg Castle

The observatory contained a variety of devices, including a large mural quadrant having a radius of 165 cm. (5.41 ft.). This quadrant was used to accurately measure a star's elevation. The quadrant was built into a wall and centered on an open window. A quadrant is similar to a sextant except that it uses a 90° arc instead of a 60° arc. Figure 14 shows an artist's illustration of a quadrant.

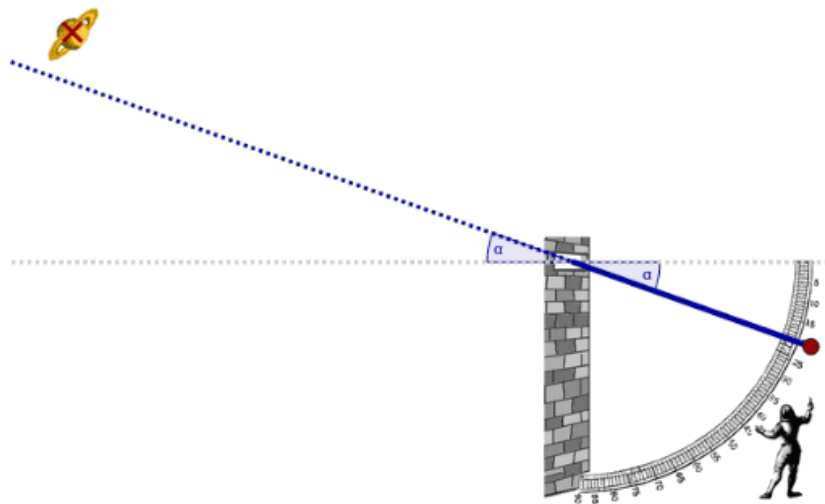


Figure 14: Example of a Quadrant

Tycho also had a number of accurate time pieces so that measurements could be made at the

same time every night. He also had the staff and equipment in order to make four simultaneous measurements of the same object. The standard accuracy of measurements prior to this time was 10 minutes of arc. Tycho was able to lower this accuracy to one minute of arc or less. One of his most trusted assistants was his sister Sophia.

In 1588 Tycho introduced his earth-centered model in which the sun and moon orbit the earth and the other planets orbit the sun. A complete description of his system was contained in his book entitled *De Mundi Aetherei Recentioribus Phaenomenis Liber Secundus* (The Second Book About Recent Phenomena in the Celestial World). In April King Frederick II died and his successor king Christian IV was less willing to continue the support of Tycho's projects. After years of negotiation Tycho had to close his observatory in 1597. He was stripped of his title as the Official Astronomer of Denmark.

Tycho didn't stay unemployed long. In 1599 he moved to Prague and was appointed "Imperial Mathematicus and Official Astrologer" by the Holy Roman Emperor Rudolf II. It was in Prague that Tycho teamed with Johannes Kepler. We will look at Kepler and his life later on.

Tycho died in October, 1601. He collapsed in pain at a party and died 11 days later. An autopsy was not able to pinpoint the cause of death. There have been many conspiracy theories claiming that Tycho didn't die of natural causes. Some even thought that he was murdered by Kepler. His body was exhumed in 1901 and examiners found traces of mercury. His body was exhumed again in 2010 and Czech scientists determined that the amount of Mercury in Brahe's body was not lethal. They concluded that Tycho died of an acute bladder infection.

## The Trial of Galileo



There were a number of factors unrelated to science that contributed to Galileo's condemnation by the church. Historically the church had been fairly tolerant towards scientific challenges, being willing to alter interpretations of scripture when it was deemed necessary. However, the Catholic church at this time was battling the rise of Protestantism and one of the main points of contention was who had the authority to interpret scripture. The Catholic church maintained that only official church theologians had this authority. Galileo had suggested that a moving earth did not contradict the scriptures since the Bible often uses metaphors when speaking of natural events. Therefore, they felt that it was dangerous at this time to allow a layman such as Galileo to dictate how certain passages of scripture should be interpreted. This may be one of the reasons that the church took a tough stand when dealing with Galileo.

In addition, Galileo made a number of political blunders. Initially the Jesuits had been strong supporters of Galileo. They had confirmed his observations with the telescope and had enthusiastically endorsed him to the officials in Rome. However, starting in 1611, Galileo began studying the motion of sunspots (small dark spots on the surface of the sun). These were observed independently by a prominent Jesuit astronomer, Christopher Scheiner. Scheiner believed that these spots were small dark objects orbiting the sun at some distance. Galileo believed that the evidence pointed to the spots being on the surface of the sun. Galileo published his results in 1613 and asserted his priority of discovery. This angered Scheiner.

In 1618 three new comets were observed. Orazio Grassi, a prominent Jesuit mathematician, wrote a book using the pseudonym Lothario Sarsi that discussed the comets. In this book he argued that comets followed paths close to those of planets, but had shorter lifetimes. Galileo believed that comets were not orbiting bodies, but were an atmospheric disturbance. In a 1623 publication, *The Assayer*, Galileo offered support for his position and made the following degrading remark concerning Sarsi

*In Sarsi I seem to discern the firm belief that in philosophising one must support oneself on the opinion of some celebrated author, as if our minds ought to remain completely sterile and barren unless wedded to the reasoning of someone else. Possibly he thinks that philosophy is a book of fiction by some author, like the Iliad or Orlando Furioso—productions in which the least important thing is whether what is written in them is true. Well, Sarsi, that is not how things are. Philosophy is written in this grand book the universe, which stands continually open to our gaze. But the book*

*cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these one wanders about in a dark labyrinth.*

This attack on one of their own further angered the Jesuits. Galileo, in his writings, would often seek to destroy his opponents as well as their arguments. It turns out that there were problems with both Galileo's and Grassi's explanations. Grassi was right that comets were orbiting objects, but their paths were elliptical and not circular. He was also wrong in his placement of the orbits. Galileo was wrong in claiming that comets were an atmospheric phenomena. However, Galileo was skilled in defending his positions even when he was wrong.

As was mentioned previously in the introduction, Galileo's biggest enemies were from the academic community. Aristotle was treated as a hero in academic circles, and Galileo's attacks on Aristotelianism were met with considerable resistance. In addition Galileo chose to publish in Italian, the language of the people, rather than in Latin as was the norm in academic publications. It is probable that some of the academics used their influence to incite the church against Galileo.

The sun-centered model of Copernicus received little attention by the church in the years following its release. This was probably due to the fact that most viewed it as a computational device and not as a representation of reality. However, the attention paid to it by Galileo caused some concern. The Copernican system was condemned by the church in 1616. The Pope asked Cardinal Bellarmine to convey news of this condemnation to Galileo. Bellarmine was the chief theologian of the church. Galileo met with Bellarmine and was given an affidavit that stated that Galileo was to no longer to hold or defend the propositions that the earth moves and the sun doesn't. Bellarmine himself didn't seem to be completely closed-minded on the subject. In a letter to a friend he once stated

*Third, I say that if there were a true demonstration that the sun is at the center of the world and the earth is in the third heaven, and that the sun does not circle the earth but the earth circles the sun, then one would have to proceed with great care in explaining the scriptures that appear contrary, and say rather that we do not understand them than what is demonstrated is false. But I will not believe such a demonstration, until it is shown me.*

This doesn't sound like a mindless rejection of the Copernican system. Unknown to Galileo, an unsigned memo was given to the Pope that supposedly came from the meeting with Cardinal Bellarmine. It stated that Galileo was no longer to hold, defend, or teach the aforementioned propositions. The inclusion of the word **teach** was important since it meant that Galileo could not even describe the Copernican system. This memo would show up later in the trial of Galileo. It is possible that this unsigned memo was produced by some of Galileo's enemies.

In 1623 Mafeo Barberini became Pope Urban VIII. Barberini was a long-time admirer of Galileo. In a conversation he once advised Galileo to not use the title *The Flux and Reflux of the Sea* for a

book he was writing as it focused to much attention on Galileo's belief that tides were due to the motion of the earth and provided evidence of the earth's motion. "After all," he declared, "God in his infinite wisdom could have created the tides in many other ways, including some beyond human intellect." In fact, it would later be shown that Galileo's theory of the tides was incorrect.

With his friend and admirer now the Pope, Galileo felt more confident in arguing for the Copernican system. In 1632 he published a document in which he disguised his position by presenting his arguments as part of a three person dialogue. It was titled **Dialogue Concerning the Two Chief Systems of the World — Ptolemaic and Copernican**. One of the participants in this dialogue was named Salviati. He was the spokesman for Galileo and presented brilliantly the case for Copernicanism. The second participant was named Simplicio. He represented an Aristotelian professor who was portrayed as ill-informed and not very bright. The name Simplicio is very close to the Italian word for simpleton. The third participant was named Sagredo. He represented an open-minded unbiased observer. Towards the end of the dialogue Galileo introduced tides as evidence for the earth's motion. He made the mistake of putting the Pope's previous remarks on tides in the mouth of Simplicio, the simpleton in this dialogue. It is unlikely that Galileo was trying to ridicule the Pope. However, Galileo's enemies picked this up quickly and convinced the Pope that he was being ridiculed. The Pope was not amused.

In August of 1632 the Inquisition in Rome issued an order to stop the publication of the Dialogue, and Galileo was ordered to stand trial. The trial was not about the scientific merits of Galileo's views, but was about whether Galileo had disobeyed an official order. The unsigned memo was issued as evidence, but Galileo denied ever receiving a copy. Three officials reviewed the Dialogue and agreed that it did advocate Copernicanism. In an unusual move, it was suggested that Galileo could get off with a lighter sentence if he would admit some wrongdoing. He agreed to remove any parts the **Dialogue** that seemed to advocate Copernicanism and he admitted that he had gone too far in some of his arguments. He was sentenced to house arrest for the rest of his life, most of which was spent in his large villa near Florence. Three of the ten Cardinals involved in the inquisition did not sign the verdict.

In the few years that remained of his life he made a number of important contributions to mechanics. Certainly the trial of Galileo was an unfortunate event that involved mistakes by both the Catholic church and by Galileo. However, to view this as a battle between science and Christianity is much too simple. There were a number of other factors involved. Science was undergoing a major paradigm shift and the church was facing significant opposition from the Protestant movement. Galileo also shares some of the blame for the way he treated those who disagreed with him. It should also be noted that Galileo himself was a committed Christian before the trial and remained one afterwards.

The paradigm shift in science forced Christian theologians to reexamine verses that seemed to support a stationary earth. Today these verses are viewed differently. For example, the verse

*Tremble before him all the earth! The world is firmly established; it cannot be moved.*

1 Chronicles 16:30

is seen as a poetic or metaphoric way of describing the stability of God's creation. Other verses

such as

*The sun rises and the sun sets, and hurries back to where it rises* Ecclesiastes 1:5

are seen as using phenomenological language. Certainly, the purpose of these verses was not to advocate some astronomical theory. The reinterpretation of these scriptures did not affect any major Christian doctrine.

# Kepler: A Better Model



Kepler (1571–1630)

The model we use today is not the one introduced by Copernicus, but is essentially the one introduced later by Johannes Kepler. Kepler was a protege of Tycho Brahe and used Tycho's measurements to arrive at his model. Kepler's model is based on the following three laws:

**Kepler's First Law** A planet moves in a plane along an elliptical orbit with the sun at one focus.

**Kepler's Second Law** The position vector from the sun to a planet sweeps out area at a constant rate.

**Kepler's Third Law** The square of the period of a planet around the sun is proportional to the cube of the average distance between the planet and the sun.

**Note on Circular Motion and uniform speed** All the models prior to Kepler's model involved objects moving in circular orbits at uniform speed. You might wonder why this restriction to circular motion and uniform motion lasted so long. The idea originated with Aristotle. He believed that all the heavenly objects that were farther from the earth than the moon were composed of a special material called aether or quintessence whose essence was to move in circular orbits at uniform speed. For a long time this was accepted as a fact of nature and wasn't questioned. It was an amazing achievement for Kepler to set aside this long held belief.

Kepler's laws were not derived from some theory, but were obtained by carefully examining empirical data. Kepler's laws were the first description of planetary orbits that didn't require epicycles to produce accurate results. Let us look briefly at the man behind these laws.

Johannes Kepler was born in the German city of Weil der Stadt on December 27, 1571. His family could be described as dysfunctional. His father was abusive and frequently left for extended periods of time to fight as a mercenary. Kepler described his mother as "sharp-tongued, quarrelsome, and possessing a bad spirit." Johannes almost died of smallpox when he was four years old and had poor health throughout his life. He also suffered from poor eyesight, a severe handicap for an astronomer. Kepler was educated in Lutheran schools and was a very good student. His family couldn't afford to send him to a university, but he received a scholarship to the University of Tübingen (see Figure 15 ) in order to pursue his goal of becoming a Lutheran pastor.



Figure 15: University of Tübingen

At the university he excelled in mathematics and astronomy in addition to theology. It was here that he first heard of the theory proposed by Copernicus and he was very intrigued. In 1594, shortly before completing his studies at the university, his career path changed abruptly. The university was asked to supply someone to fill a vacant position of a mathematics and astronomy teacher in a protestant boys school in Graz, Styria (a district of Austria). Kepler was chosen by the university to fill this position. He was not pleased by this move, but gradually he realized that this was part of God's plan. Here is what he was later quoted as saying

*I had the intention of becoming a theologian... but now I see how God is, by my endeavors, also glorified in astronomy, for 'the heavens declare the glory of God.'*

His teaching position in Graz gave him the opportunity to further explore his interest in astronomy

and the Copernican theory. In 1596 he published his first paper in astronomy entitled *Mysterium Cosmographicum*. In this paper he tried to show that the spacing between the various planets could be explained geometrically in terms of the five regular polyhedra. This was not very successful, but in this paper he made an observation that would later have great importance. He observed that the planets moved faster when they were close to the sun and slower when they were farther from the sun. He postulated that there was some unknown force emanating from the sun that influenced the motion of the planets. This idea was later pursued by Newton.

In 1600 Kepler was forced to leave Graz as the Catholic leadership there decided to expel all protestants. Kepler had previously collaborated informally with Tycho Brahe, but he now accepted a position as Tycho's assistant. Tycho was well known for his accurate astronomical measurements. He was assigned the task of coming up with a better description of the orbit of Mars. One of the first things that Kepler did was to actually plot the motion of Mars as predicted by the Ptolemaic model. This motion is shown in Figure 16.

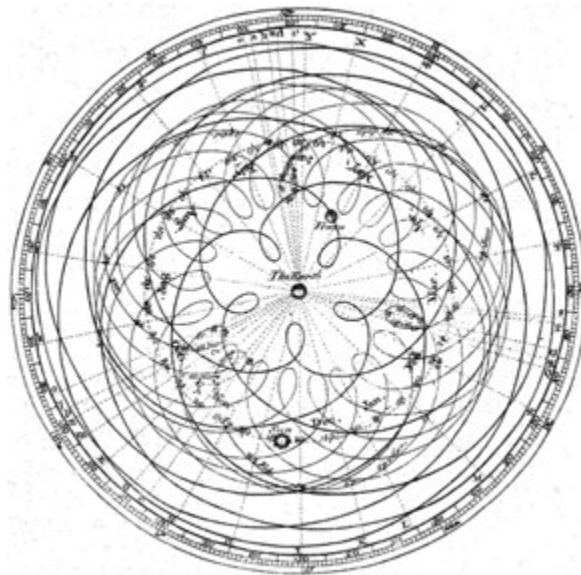


Figure 16: Plot of Mars orbit as predicted by Ptolemaic model. Taken from *Astronomia nova*, chapter 1 (1609)

Kepler found it hard to believe that Mars would follow such a strange path with all the loops. He was attracted to the Copernican model, but he was not satisfied with the accuracy of its results.

When Tycho died in 1601, Kepler became his successor. In 1603 Kepler interrupted his research on Mars orbit to produce a book on optics called *Astronomiae Pars Optica*. In this book he describes such things as the inverse square law for the intensity of light, atmospheric refraction, and parallax. He also describes the optics of the human eye.

When Kepler returned to the study of Mars orbit he used Tycho's measurements to come up with his first two laws of planetary motion. He actually came up with the second law first and later determined that the orbit must be elliptical with the sun at a focus. In 1609 he published his results in a paper entitled *Astronomia nova*. The third law was not discovered until 1618. A more complete

exposition of his laws was entitled *Epitome astronomiae copernicanae*. It was published in three parts between 1618 and 1621.

In 1611 Kepler published a paper entitled *dioptrice* that explained the optics of a telescope. In preparing this paper he came up with a better design for the telescope. The telescopes at this time used a convex lens to capture the light and a concave eyepiece to focus the image. Kepler designed a telescope in which both of the lens were convex. This had a slight disadvantage in that the image was inverted. However, this was overshadowed by the fact that Kepler's telescope had a much larger field of view.

In 1621 Kepler published a set of tables called the Rudolphine tables that were based on his three laws. These tables allowed for the future prediction of the position of the stars and planets. Kepler died rather suddenly in 1630 during a trip to Regensburg. This was about two years before Galileo's trial. He was buried somewhere outside the city, but all the graves there were destroyed a couple years later during the 30 years war.

It should be noted that Galileo and Kepler were contemporaries and corresponded frequently. Although Galileo was aware of Kepler's work he never abandoned his belief in circular orbits. Even though Kepler's model was a major advancement, it was slow to catch on. For one thing, his publications were overshadowed by those of Galileo that were much more widely distributed. In addition, Galileo's advocacy of the Copernican model prompted the Catholic church to take a hard line on publications promoting a sun-centered view. Thus, Kepler's *Astronomia nova* as well as some of his later works were placed on the index of banned books. It was the success of his Rudolphine tables that sparked interest in his model. Some astronomers accepted his arguments for elliptical orbits, but there was no general acceptance of his full model. For one thing, Kepler's second law proved to be very difficult to apply in practice since it doesn't provide a direct relationship between the position of a planet and time.

In 1631 the French astronomer Pierre Gassendi observed the transit of Mercury across the face of the sun that was predicted by Kepler's Rudolphine tables. This was the first time this transit had been seen. In 1639 Jeremiah Horrox was able to observe the transit of Venus. These observations were evidence for the validity of Kepler's laws. In 1687 Kepler's laws were derived by Newton from his laws of motion and his law of universal gravitation. This proved to be the key to the general acceptance of Kepler's model. The derivation of Kepler's laws from Newton's laws is contained in the Appendix. The final nail in the coffin of the earth-centered view was the measurement of stellar parallax. In 1838 Friedrich Bessel made the first successful parallax measurement, for the star 61 Cygni, using a Fraunhofer heliometer at Königsberg Observatory. Over 6 months he detected a slight angular shift of 0.3 seconds or 83 millionths of a degree.

# Role in Science History

In this section we will look at the role that the change from the earth-centered view of Aristotle to the sun-centered view played in the history of science. In the process we will gain more insight into the nature of science and how it works.

Most historians of science agree that what we call modern science originated in Europe during the 16th century. This is somewhat surprising since the European society at that time was not nearly as advanced as the Greek, Islamic, and Chinese cultures of the past. The question then is “Why Europe?” The main thing that separates science from previous approaches to the study of the natural world is the emphasis on observation and experiment. The Greeks, for example, relied primarily on reason and placed little value on experiment. The emphasis on observation arose, at least in part, from the Christian Faith that was dominant in Europe at this time. A central Christian belief is that God created the heavens and the earth starting from nothing. Thus, God was free to design any kind of universe that he desired. It follows that logic alone cannot discover what was in the mind of God. To discover the nature of God’s creation, it was necessary to look and see, i.e., observe and experiment.

The story presented in this paper took place in the early days of the scientific revolution. We can see the importance that observational data played in replacing the earth-centered views of Aristotle by a sun-centered view. It was Galileo’s observations with his telescope that exposed some of the problems with the Aristotelian view. It was Tycho Brahe’s accurate astronomical data that led Kepler to his laws of planetary motion. Much later, it was Bessel’s measurement of stellar parallax that firmly established the sun-centered view.

The story presented here also shows that observational and experimental data are not neutral, but can be influenced by philosophical considerations, i.e., the observer’s worldview. Although many aspects of Aristotle’s view of the natural world were based on reason rather than observation, there were some aspects of his viewpoint that seemed to agree well with our observations, e.g., the fact that we don’t feel any of the effects (such as air flow) that are normally associated with motion. In addition, there was the failure to observe stellar parallax and the difficulty in moving massive objects. It was these observations that delayed the acceptance of a sun-centered view. It was not until Galileo, Kepler, and Newton provided a different conceptual framework that the observations that seemed to support an earth-centered view could be interpreted differently.

The history presented here also illustrates the interplay of data and theories. Theories are based on certain assumptions or hypotheses. Predictions are then derived from these hypotheses that

are tested against experimental or observational data. It is rare that the hypotheses themselves can be tested directly, but it is predictions based on these hypotheses that are tested. If a theory consistently makes verifiable predictions, it is often labeled a law. Kepler used his laws to make tables of planetary predictions that were later confirmed by observation.

Scientists, like everyone else, have their biases, and live in a society where there are shared beliefs about the natural world and how to study it. We like to think of science as dealing with purely factual data, but bare facts have little use without some interpretation. Throughout history, the major advances in science have not depended so much on new or better data, but on new ways of viewing the data: new conceptual frameworks.

You can also see from this example that major changes in science often require a considerable amount of time and usually face considerable resistance prior to their acceptance. I hope this gives you a better appreciation of the important role that this piece of history played in the advancement of science.

# Apendices

# Kepler's Laws Derived from Newton's Laws

In this appendix we will derive Kepler's laws from Newton's laws of motion and his law of universal gravitation.

**Kepler's laws** Below are the three laws that were derived empirically by Kepler.

- Kepler's First Law: A planet moves in a plane along an elliptical orbit with the sun at one focus.
- Kepler's Second Law: The position vector from the sun to a planet sweeps out area at a constant rate.
- Kepler's Third Law: The square of the period of a planet around the sun is proportional to the cube of the semi-major axis length.

**Mathematical preliminaries** Consider a Cartesian coordinate system with the sun at the origin. Let  $(x, y, z)$  denote the position of a planet. Clearly  $x$ ,  $y$ , and  $z$  are functions of the time  $t$ . We define the position vector  $\mathbf{r}$ , the velocity vector  $\mathbf{v}$ , and the acceleration vector  $\mathbf{a}$  by

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \quad \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}, \quad \mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$

Here the dots represent differentiation with respect to time and  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  are the unit vectors in the  $x$ ,  $y$ ,  $z$  directions respectively. Newton's law of motion can be written

$$\mathbf{F} = m\mathbf{a} \tag{1}$$

where  $m$  is the mass of the planet and  $\mathbf{F}$  is the force on the planet. Let  $\hat{\mathbf{r}}$  be a unit vector in the  $\mathbf{r}$  direction. Then Newton's law of gravitation applied to the earth and sun is given by

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}} = -\frac{GMm}{r^3}\mathbf{r} \tag{2}$$

where  $G$  is a constant,  $M$  is the mass of the sun, and  $r$  is the magnitude of  $\mathbf{r}$ . Here we have assumed that the sun is at the origin and that  $\mathbf{r}$  is the position vector of the planet. We will neglect the gravitational forces due to the other planets. Combining equations (1) and (2), we get

$$\mathbf{a} = \ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} \tag{3}$$

**Planet moves in a plane** By the product rule for differentiation

$$\frac{d}{dt}(\mathbf{r} \times \mathbf{v}) = \mathbf{v} \times \mathbf{v} + \mathbf{r} \times \mathbf{a} = 0$$

since  $\mathbf{a}$  is in the same direction as  $\mathbf{r}$  by equation (3). Here the symbol  $\times$  represents the vector cross-product. Thus, the vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

is a constant. It follows that  $\mathbf{r}$  and  $\mathbf{v}$  lie in the plane orthogonal to  $\mathbf{h}$ . We will choose our coordinate system so that  $\mathbf{k}$  is in the direction  $\mathbf{h}$ . Thus,

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} = hk \quad \text{where } h > 0. \quad (4)$$

**Kepler's second law** Figure 17 shows the area swept out by the position vector in a small increment of time  $\Delta t$ .  $\Delta\theta$  is the small change of angle.

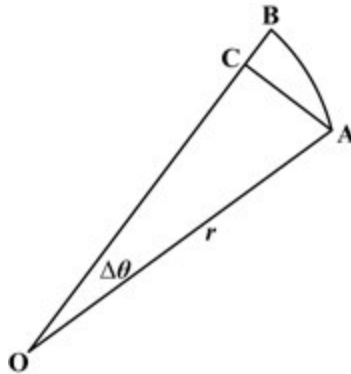


Figure 17: Area swept out during small time increment

The area OAB is approximately equal to the area of the right triangle OAC for small  $\Delta\theta$ . Since the length of the line AC is approximately  $r\Delta\theta$  and the length of the line OC is approximately  $r$ , we have

$$\Delta A \doteq \frac{1}{2}r^2\Delta\theta.$$

Dividing by  $\Delta t$  and letting the time increment approach zero, we see that

$$\dot{A} = \frac{1}{2}r^2\dot{\theta}. \quad (5)$$

Since the planet moves in the  $xy$  plane, we have

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j} \quad (6)$$

where the polar coordinates  $r$  and  $\theta$  are functions of  $t$ . The time derivative of  $\mathbf{r}$  is given by

$$\mathbf{v} = (\dot{r} \cos \theta - r \sin \theta \dot{\theta})\mathbf{i} + (\dot{r} \sin \theta + r \cos \theta \dot{\theta})\mathbf{j}. \quad (7)$$

Substituting equations (6) and (7) into equation (4), we obtain

$$h = r \cos \theta (\dot{r} \cos \theta + r \sin \theta \dot{\theta}) - r \sin \theta (\dot{r} \cos \theta - r \sin \theta \dot{\theta}) = r^2 \dot{\theta}. \quad (8)$$

Here we have used the fact that  $\mathbf{i} \times \mathbf{j} = \mathbf{k}$  and  $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$ . It follows from equations (5) and (8) that

$$\dot{A} = \frac{1}{2} r^2 \dot{\theta} = h/2 = \text{constant}.$$

This is Kepler's second law.

**Definition and properties of an ellipse** Before we look at the derivation of Kepler's first law, we need to define what we mean by an ellipse, and look at some of its properties. One common way of drawing an ellipse is to pin the two ends of a string, place a pencil in the loop, and trace a curve while keeping the string taut. Clearly the resulting curve has the property that the sum of the distances from any point on the curve to the two fixed points is a constant (the length of the string). The resulting curve is called an ellipse and the two fixed points are called the foci of the ellipse. Figure 18 shows an ellipse in which the foci are at  $(-c, 0)$  and  $(c, 0)$ , and  $2a$  corresponds to the length of the string.

The construction of the ellipse can be represented mathematically as follows

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a \quad \text{where } a > c > 0 \quad (9)$$

This equation can be rearranged as follows

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}.$$

Squaring both sides, we get

$$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2.$$

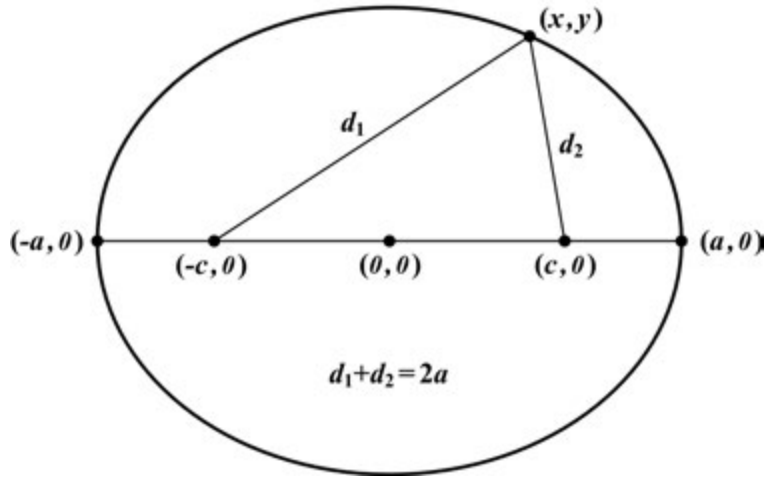


Figure 18: Drawing of an ellipse

Solving for the square root term, we obtain

$$\sqrt{(x-c)^2 + y^2} = \frac{1}{4a}[4a^2 + (x-c)^2 - (x+c)^2] = a - \frac{c}{a}x.$$

Squaring again, we obtain

$$x^2 - 2cx + c^2 + y^2 = a^2 - 2cx + \frac{c^2}{a^2}x^2$$

or equivalently

$$\left(1 - \frac{c^2}{a^2}\right) + y^2 = (a^2 - c^2)\frac{x^2}{a^2} + y^2 = a^2 - c^2.$$

Dividing through by  $a^2 - c^2$ , we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \quad (10)$$

We define the *eccentricity*  $e$  of the ellipse by  $e = c/a$ . The eccentricity is a measure of the elongation of the ellipse. The eccentricity of the earth's orbit is small (.0167). Thus, its orbit is nearly circular. Venus has an even smaller eccentricity (.007) and Mars has a larger eccentricity (.0934). The planet with the largest eccentricity is Mercury (.2056). Let us define  $b$  by

$$b = a\sqrt{1 - e^2} = \sqrt{a^2 - c^2} \quad (11)$$

Thus, equation (10) can be written in the standard form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (12)$$

This is the form that is usually specified for an ellipse. It is easy to see that  $a$  is one-half the length of the ellipse's major axis and  $b$  is one-half the length of the ellipse's minor axis.

An ellipse also has a simple form in polar coordinates if we take our origin to be one of the foci. This situation is pictured in Figure 19.

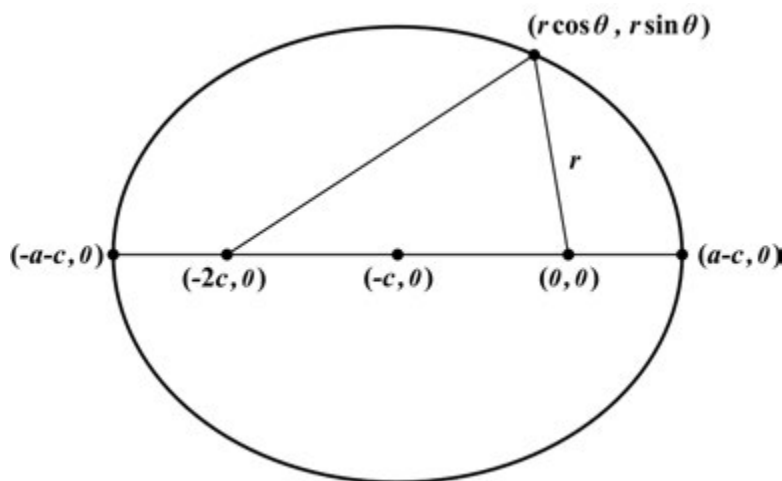


Figure 19: An ellipse in polar coordinates

Using the definition of an ellipse in terms of the sum of the distances from the two foci being constant, we can write

$$r + \sqrt{(r \cos \theta + 2c)^2 + r^2 \sin^2 \theta} = 2a. \quad (13)$$

Solving for the square root term and expanding the square terms, we get

$$\sqrt{r^2 + 4rc \cos \theta + 4c^2} = 2a - r.$$

Squaring this equation gives

$$r^2 + 4rc \cos \theta + 4c^2 = 4a^2 - 4ar + r^2$$

or equivalently

$$(a + c \cos \theta)r = a^2 - c^2.$$

Solving for  $r$ , we obtain

$$r = \frac{a^2 - c^2}{a + c \cos \theta} = \frac{a^2(1 - \frac{c^2}{a^2})}{a(1 + \frac{c}{a} \cos \theta)} = \frac{a(1 - e^2)}{1 + e \cos \theta} = \frac{k}{1 + e \cos \theta} \quad (14)$$

where  $k = a(1 - e^2)$ . Equation (14) is the desired representation of the ellipse in polar coordinates.

We can also derive our original definition of an ellipse from the polar form. Suppose  $r$  and  $\theta$  satisfy

$$r = \frac{k}{1 + e \cos \theta} \quad \text{where } k > 0 \text{ and } 0 < e < 1. \quad (15)$$

We define  $a$  and  $c$  by

$$a = \frac{k}{1 - e^2} = \frac{k}{(1 - e)(1 + e)} \quad \text{and} \quad c = ae. \quad (16)$$

It follows from equation (15) that  $r$  has a maximum value of  $\frac{k}{1 - e}$  at  $\theta = \pi$ . Thus,

$$r \leq \frac{k}{1 - e} = a(1 + e) < 2a.$$

Equation (15) can be rearranged as follows

$$(1 + e \cos \theta)r = k = a(1 - e^2).$$

Since  $e = c/a$ , this equation can be written

$$\left(1 + \frac{c}{a} \cos \theta\right)r = a\left(1 - \frac{c^2}{a^2}\right) = \frac{a^2 - c^2}{a}.$$

Multiplying both sides by  $a$ , we obtain

$$(a + c \cos \theta)r = a^2 - c^2.$$

Multiplying this equation by 4 and adding  $r^2 = (\sin^2 \theta + \cos^2 \theta)r^2$  to both sides, we obtain

$$(\cos^2 \theta + \sin^2 \theta)r^2 + 4ar + 4cr \cos \theta = 4a^2 - 4c^2 + r^2.$$

This equation can be rearranged as

$$r^2 \cos^2 \theta + 4cr \cos \theta + 4c^2 + r^2 \sin^2 \theta = r^2 - 4ar + 4a^2.$$

or equivalently

$$(r \cos \theta + 2c)^2 + r^2 \sin^2 \theta = (2a - r)^2.$$

Taking the square root of both sides, we obtain

$$r + \sqrt{(r \cos \theta + 2c)^2 + r^2 \sin^2 \theta} = 2a$$

which is the defining equation for the ellipse pictured in Figure 19 [see equation (13)]. Thus, equation (15) defines an ellipse with the origin at one focus. Let  $b = a\sqrt{1 - e^2}$ . Then it follows from equation (16) that

$$b = \frac{k}{\sqrt{1 - e^2}} \quad (17)$$

**Hamilton's Theorem** In this section we will show that the velocity vector  $\mathbf{v}$  moves on a circle. Since  $r = |\mathbf{r}|$ , equation (3) can be written

$$\dot{\mathbf{v}} = \mathbf{a} = -\frac{GM}{r^2}(\cos \theta \mathbf{i} + \sin \theta \mathbf{j}). \quad (18)$$

Combining equations (8) and (18), we obtain

$$\dot{\mathbf{v}} = -\frac{GM}{h} \dot{\theta} (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}). \quad (19)$$

By the chain rule for differentiation

$$\dot{\mathbf{v}} = \frac{d\mathbf{v}}{d\theta} \dot{\theta}. \quad (20)$$

It follows from equations (19) and (20) that

$$\frac{d\mathbf{v}}{d\theta} = -\frac{GM}{h}(\cos \theta \mathbf{i} + \sin \theta \mathbf{j}).$$

Integrating this equation, we obtain

$$\mathbf{v} = \frac{GM}{h}(-\sin \theta \mathbf{i} + \cos \theta \mathbf{j}) + \mathbf{v}_0 \quad (21)$$

where  $\mathbf{v}_0$  is a constant. It follows that  $|\mathbf{v} - \mathbf{v}_0| = GM/h$ , i.e.,  $\mathbf{v}$  moves on the circle centered at  $\mathbf{v}_0$  with radius  $GM/h$ .

**Kepler's first law** We choose our coordinate system so that  $\mathbf{j}$  is in the direction  $\mathbf{v}_0$ , i.e.,

$$\mathbf{v}_0 = v_0 \mathbf{j} \quad \text{where } v_0 > 0. \quad (22)$$

Thus, equation (21) becomes

$$\mathbf{v} = \frac{GM}{h}[-\sin \theta \mathbf{i} + (\cos \theta + e) \mathbf{j}] \quad (23)$$

where  $e = v_0 h / GM$ . Substituting equation (23) into equation (4) and using equation (6), we get

$$h\mathbf{k} = \mathbf{r} \times \mathbf{v} = \frac{GMr}{h}[\sin^2 \theta + (\cos^2 \theta + e \cos \theta)] = \frac{GMr}{h}(1 + e \cos \theta)\mathbf{k}.$$

and hence

$$r = \frac{h^2}{GM} \frac{1}{1 + e \cos \theta} = \frac{k}{1 + e \cos \theta} \quad (24)$$

where  $k = h^2 / GM$ . In order for  $r$  to remain finite for all  $\theta$ , we must have  $0 \leq e < 1$ . Equation (24) is the equation of an ellipse in polar coordinates with the origin at one focus. This completes the proof of Kepler's first law.

**Kepler's third law** Since the rate that area is swept out by the position vector is the constant  $h/2$ , it follows that

$$A = hT/2 \quad (25)$$

where  $T$  is the period of the motion and  $A$  is the area of the ellipse. Since translation doesn't change the area, we can consider the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (26)$$

We will calculate the area of the first quadrant ( $x \geq 0, y \geq 0$ ) and multiply by 4. Solving for  $y$  as a function of  $x$  from equation (26), we obtain

$$y = b\sqrt{1 - x^2/a^2}, \quad 0 \leq x \leq a.$$

Thus, the area  $A$  is given by

$$A = 4b \int_0^a \sqrt{1 - x^2/a^2} dx. \quad (27)$$

If we make the change of variables  $x = a \sin \phi$  ( $dx = a \cos \phi d\phi$ ) in the integral, we obtain

$$A = 4ab \int_0^{\pi/2} \cos^2 \phi d\phi = 4ab \int_0^{\pi/2} \frac{1 + \cos 2\phi}{2} d\phi = \pi ab. \quad (28)$$

Substituting this value for  $A$  into equation (25), we obtain

$$T = \frac{2\pi ab}{h} \quad \text{and hence} \quad T^2 = \frac{4\pi^2 a^2 b^2}{h^2}. \quad (29)$$

Using equations (16) and (17) along with the relation  $k = h^2/GM$ , we can write the expression for  $T^2$  in equation (29) as follows

$$T^2 = \frac{4\pi^2 k^4}{(1 - e^2)^3 h^2} = \frac{4\pi^2 k a^3}{h^2} = \frac{4\pi^2 a^3}{GM}. \quad (30)$$

Equation (30) will establish Kepler's third law if we can show that  $a$  is the average distance between a point on the ellipse and the focus where the sun is located. The distance  $D$  of a point  $(x, y)$  on the ellipse to the focus  $(c, 0)$  is given by

$$D = \sqrt{(x - c)^2 + y^2}. \quad (31)$$

It follows from equation (12) that

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right).$$

Combining this equation with equation(31), we get

$$\begin{aligned}
 D &= \sqrt{(x-c)^2 + b^2 \left(1 - \frac{x^2}{a^2}\right)} \\
 &= \sqrt{x^2 \left(1 - \frac{b^2}{a^2}\right) - 2cx + c^2 + b^2}.
 \end{aligned} \tag{32}$$

It follows from equation(11) that

$$1 - \frac{b^2}{a^2} = \frac{c^2}{a^2} \quad \text{and} \quad c^2 + b^2 = a^2.$$

Using these relations, equation (32) becomes

$$\begin{aligned}
 D &= \sqrt{\frac{c^2}{a^2}x^2 - 2cx + a^2} \\
 &= \sqrt{\frac{(cx - a^2)^2}{a^2}}.
 \end{aligned} \tag{33}$$

Since  $x \leq a$  and  $c < a$ , it follows that  $cx < a^2$ . Thus

$$D = \frac{a^2 - cx}{a}. \tag{34}$$

For the upper half of the ellipse the average distance  $D_{\text{av}}$  is given by

$$\begin{aligned}
 D_{\text{av}} &= \frac{1}{2a} \int_{-a}^a \frac{a^2 - cx}{a} dx \\
 &= \frac{1}{2} \int_{-a}^a \left(1 - \frac{c}{a^2}x\right) dx \\
 &= a
 \end{aligned} \tag{35}$$

since

$$\int_{-a}^a x dx = 0.$$

The average distance over the lower half of the ellipse is the same; therefore, equation (35) represents the average distance over the ellipse. Equations (30) and (35) combine to give Keplers third law.

## Galileo and sun spots

Although sun spots had been observed with the naked eye as far back as 28 AD, Galileo and Thomas Harriot were the first to observe them with the telescope in the last part of 1610. Galileo didn't study them in detail at this time, but Christoph Scheiner, a Jesuit Mathematician, undertook a study in March of 1611. Scheiner accepted the Aristotelian viewpoint that the sun must be a perfect body and argued that the spots were due to a satellite of the sun passing in front of it. However, Galileo later argued that the spots were on the sun itself. Galileo observed the sun spots over time by projecting the telescope image onto a piece of paper and coloring the spots with a pen. Figure 20 shows one of his drawings.

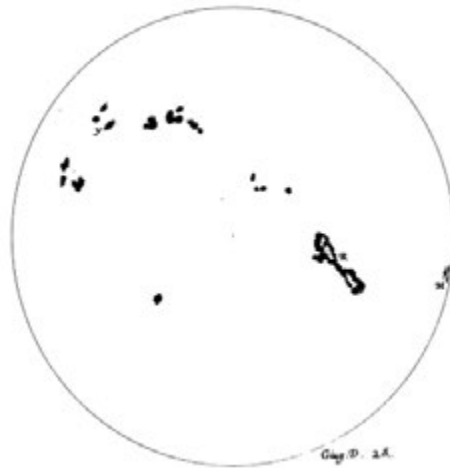


Figure 20: Galileo Drawing of Sun Spots

Galileo gave several arguments for his claim that the sun spots were on the surface of the sun.

- The spots were irregular in shape whereas planets tended to be more circular.
- Sometimes the spots appeared for the first time inside the sun's image and disappeared before crossing the image.
- The spots that appear near the edge of the image were thinner and moved slowly. As they moved across the image they grew in size and moved faster. This is consistent with the spots being on the sun and the sun rotating. Near the edge the spots are moving towards you and not moving much across the image. There is no reason that an orbiting satellite would produce a spot that changes greatly in speed as it crosses the image.

For a long time Scheiner was angry with Galileo for not giving him credit for his pioneering work in this area. Eventually Scheiner agreed with Galileo and gave up the Aristotelian viewpoint for Tycho Brahe's model.

# References

- [1] DeWitt, Richard, **Worldviews: An Introduction to the History and Philosophy of Science**, Second Edition, Wiley-Blackwell (2010).
- [2] Shea, William and Artigas, Mariano, **Galileo in Rome: The Rise and Fall of a Troublesome Genius**, Oxford University Press (2003).
- [3] Crowe, Michael J., **Theories of the World: from Antiquity to the Copernican Revolution**, Second Revised Edition, Dover (2001).
- [4] Newall, Paul, *The Galileo Affair*, available at [Galileo Affair](#).
- [5] Fowler, Michael, *Galileo and Einstein*, available at [Galileo and Einstein](#).
- [6] Robbin, Joel W., *Kepler's Laws*, available at [Kepler's Laws](#).
- [7] Love, David K., **Kepler and the Universe**, Prometheus Books (2015).
- [8] Voelkel, James R., **Johannes Kepler and the New Astronomy**, Oxford University Press (1999).
- [13] Ferguson, Kitty, **Tycho & Kepler: The Unlikely Partnership That Forever Changed Our Understanding of the Heavens**, Walker, New York, 2002.
- [13] Charles River Editors, **Tycho Brahe and Johannes Kepler: The Lives of the Early Modern Astronomers Whose Partnership Revolutionized the Field of Astronomy**, Charles River Editors (2018).
- [13] Ball, Robert S., *Great Astronomers*, 1895. Available online at [Great Astronomers](#)
- [13] Chaman, Allan, *Tycho Brahe—Instrument designer, observer, and mechanician*, J. Br. Astron. Assoc., **99**, 1989. Available online at [Harvard](#).
- [13] Gingerich, Owen, *Galileo, the Impact of the Telescope, and the Birth of Modern Astronomy*, Proceedings of the American Philosophical Society (2011)