

James Clerk Maxwell

His Life and His Faith

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Introduction



James Clerk Maxwell (1831–1879)

In the history of physics there are three individuals who stand out above all others. They are Isaac Newton, James Clerk Maxwell, and Albert Einstein. Each of these men dramatically changed the way we look at the physical world — Isaac Newton with his laws of motion, James Clerk Maxwell with his electro-magnetic equations, and Albert Einstein with his theories of relativity. The names of Newton and Einstein are well known to the general public, but for some reason James Clerk Maxwell is not well known outside scientific circles. This is unfortunate since many of the devices we depend on today can be traced back to his pioneering work. These include radios, televisions, microwave ovens, and cell phones among many others.

Maxwell's equations united all the prior results of Faraday, Volta, Ampère, Oersted, Henry and others in electricity and magnetism. Maxwell not only developed these fundamental equations, but used them to predict the existence of electromagnetic waves (radio and TV waves for example) and to show that light is an electromagnetic wave. Maxwell's prediction of electromagnetic waves is one of those rare cases where theoretical prediction actually precedes observation. These waves were first produced and detected by Hertz more than twenty years after they were predicted theoretically by Maxwell.

The work of Maxwell has certainly not gone unnoticed by physicists. Here are some quotes by other famous scientists on the importance of James Clerk Maxwell in the history of science:

This change in the conception of reality is the most profound and the most fruitful that physics has experienced since the time of Newton [Referring to the contributions of James Clerk Maxwell to physics]. — Albert Einstein

One scientific epoch ended and another began with James Clerk Maxwell. — Albert Einstein

From a long view of the history of mankind — seen from, say, ten thousand years from now — there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade. — Richard Feynman

Maxwell's importance in the history of scientific thought is comparable to Einstein's (whom he inspired) and to Newton's (whose influence he curtailed). — Ivan Tolstoy

Maxwell's Equations have had a greater impact on human history than any ten presidents. — Carl Sagan

The contributions of Maxwell were not confined to electricity and magnetism. He was the first to develop a mathematical theory of color. In this connection he produced the first color photograph. He pioneered the use of statistics in physical theories (kinetic theory of gases) and wrote a prize winning essay on the stability of Saturn's rings. He also made fundamental contributions to thermodynamics and control theory. In the later part of his career Maxwell developed and was the first director of the Cavendish Laboratory at Cambridge. This laboratory has presently produced 29 Nobel prize winners including J.J. Thomson (discoverer of the electron) and Francis Crick (DNA).

Maxwell was greatly loved by his friends and associates. He was admired for his gentle manner, his concern for others, and his integrity. He also had a delightful sense of humor and enjoyed pulling pranks on his friends. Maxwell possessed a deeply rooted Christian faith that played a big part in both his science and his relations with others. He knew large portions of the Bible for memory and led daily prayer with his servants and staff. He was raised in the Presbyterian church, but also attended the Anglican church with his aunt. In his university years he often attended Baptist services. In later years he became an elder in the Presbyterian church.

The accomplishments of Maxwell are all the more amazing when you consider that he died at age 48. In the sections that follow we will present a brief biography of his life and look at his scientific accomplishments and his faith in more detail. In preparing this paper I looked at a great number of references, many of which are available online. Those that I found most helpful are listed in the Reference section at the end.

Biographical Sketch

James Clerk Maxwell was born in Edinburgh Scotland on 13 July 1831, the only son of John and Frances. His father, John, was a lawyer in Edinburgh who also dabbled in science. In fact, he was a fellow of the Royal Society of Edinburgh. His father's given name was John Clerk, but he inherited a 1500 acre estate in southern Scotland that had been passed down through the Maxwell family. One of the stipulations attached to this estate was that the owner must be a Maxwell. Therefore, John attached Maxwell to his name and became John Clerk Maxwell. His son then became James Clerk Maxwell. A picture of Maxwell's birthplace is shown in figure 1.



Figure 1: Maxwell's birthplace in Edinburgh

After James was born the family moved from Edinburgh to a newly built house on the estate called "Glenlair". A picture of Glenlair is shown in figure 2.



Figure 2: Maxwell's home Glenlair

As a child James was very curious. His favorite question was “What’s the go o’ that? What does it do?” If the answer didn’t satisfy him, he would come back with “But what’s the particular go of it?” When he became literate he enjoyed reading and creating poetry and fanciful short stories. He also enjoyed the outdoors and became an excellent swimmer and diver as well as a skilled ice-skater.

Even at an early age James showed a remarkable memory. At eight he could recite long passages of Milton and the entire 119th Psalm (176 verses). He could also give the chapter and verse for almost any passage from the psalms. Since they lived in a rural area, James was home schooled by his mother until her death in 1839. James was eight at the time. Although they were always close, James and his father became much closer following his mothers death.

For a short while after his mother’s death, James was taught at home by a tutor, but this didn’t work out. His father then sent him to Edinburgh Academy. This was a preparatory school giving emphasis to mathematics, natural philosophy, Latin, Greek, ethics, and the classics. While attending this school in Edinburgh he lived with his aunt, Mrs. Wedderburn. A picture of Edinburgh Academy is shown in figure 3.

Because of his rural background, James did not fit in well at first. He was often teased because of his dress and speech. For the first two years his academic performance was not exceptional. But then he began showing promise. At age 13 he won the mathematics medal and received the first prize for English and Poetry. At age 14 he published his first paper entitled *On the description of Oval Curves, and those having a plurality of foci* in the Proceedings of the Royal Edinburgh Society. Although he didn’t realize it at the time, this paper was an extension of some studies carried out by René Descartes in the seventeenth century.

While at the academy James met two lifelong friends — Lewis Campbell (a future Professor of Classics at St Andrews University) and Peter Guthrie Tait (a future Professor of Natural Philosophy at Edinburgh University). Tait and Maxwell had similar interests and discussed each other’s work throughout their lifetime. Campbell was probably Maxwell’s most intimate friend and later became his biographer.

At age 16, Maxwell’s father enrolled him in the University of Edinburgh. A picture of the University is shown in figure 4. During his three years there he divided his time between Edinburgh and Glenlair. While at Glenlair he built a home laboratory out of materials that were available. He carried out many experiments on electromagnetics and chemistry in this lab. While at the University he studied, among other things, Physics and Philosophy. He made a big impression on Professor James D. Forbes, a physicist at the university, and they became close friends. Forbes allowed Maxwell to use the physics laboratory to carry out his own experiments, and Maxwell assisted Professor Forbes in other experiments. One of the areas they worked on together was color perception. Maxwell would return to this topic later in his career. He also had two more papers accepted by the Edinburgh Royal Society. The first was on Rolling Curves, and the second was on the Equilibrium of Elastic Solids.

Maxwell’s father desired for him to follow in his footsteps and pursue a legal career. This may seem strange to us, given Maxwell’s ability in science and mathematics, but we must remember



Figure 3: Edinburgh academy



Figure 4: The University of Edinburgh

that science was not a promising career in those days. Most of those involved in science had independent sources of income or were members of the clergy. In fact, those involved in science were not called scientists, but natural philosophers. Most universities at this time had only one professor of science or natural philosophy on their faculty. However, after three years at the University of Edinburgh, James' father consented for him to go to Cambridge University, the foremost British scientific institution.

He left Scotland for Cambridge in 1850. Maxwell's undergraduate days at Cambridge began at Peterhouse college, the oldest college at Cambridge. Peterhouse attracted some of the best mathematical talent. Before the end of his first term, Maxwell transferred to Trinity college (see figure 5). This move was probably influenced by Professor Forbes who was a close friend of the master, William Whewell. Peterhouse was a small college and had limited funds for graduate fellowships whereas Trinity college was much larger and provided a better opportunity for obtaining a fellowship after graduation. Trinity had a good reputation in science, but also excelled in many fields outside of science. This fit well with Maxwell's wide range of interests and abilities.



Figure 5: Trinity college at Cambridge

Towards the end of his undergraduate days at Cambridge James finished second in the difficult mathematical tripos exams and tied for first in the prestigious Smith Prize examination. He graduated in 1854 and, as expected, received a graduate fellowship to continue his work. Shortly after graduation he published two important papers. The first was entitled *On the Transformation of Surfaces by Bending*, and the second was entitled *On Faradays Lines of Force*. The latter paper was the beginning of his work on electricity and magnetism that eventually led to his celebrated equations.

In 1856 Maxwell returned to Scotland to accept the chair of Natural Philosophy at Marischal College, Aberdeen. One of the primary reasons Maxwell left Cambridge to take this position was to be closer to his father who was seriously ill. Unfortunately, Maxwell's father died shortly before his arrival at Aberdeen.

By all accounts Maxwell was not a good classroom lecturer. His active mind often caused him

to depart from the lesson to explore tangential issues. In the process he usually lost most of the students. However, after class, a few of the more dedicated students would gather around him and benefit from his wisdom and insights. Maxwell enjoyed sharing in this way with his students. In 1858 Maxwell married Katherine Mary Dewar, daughter of the principal of Marischal College. A picture of Maxwell and his wife is shown in figure 6.



Figure 6: Maxwell and his wife Katherine

In 1859 James won the prestigious Adams prize that is offered every three years to a former Cambridge graduate. His winning essay was on the stability of Saturn's rings. In the essay he showed that the rings could not be continuous solids or fluids, but must consist of a large number of independently orbiting particles. This was verified by Voyager flybys in the 1980s. Maxwell's essay was so detailed and convincing that the English mathematician and astronomer George Airy wrote

It is one of the most remarkable applications of mathematics to physics that I have ever seen.

In 1860 Marischal college merged with neighboring Kings College and Maxwell's position was eliminated. Later that year he accepted the position of professor of Natural Philosophy at Kings College, London. Maxwell contracted smallpox shortly before assuming his new position and nearly died. He credited his recovery to Katherine's loving care. Maxwell's time in London was very productive. In 1860 Maxwell received the Royal Society's Rumford Medal for his work on the perception of color and color blindness. He also demonstrated the first color photograph by taking a photograph through three filters (red, green, and blue) and then projecting the images through three projectors having the same three filters. Later that year he was elected to membership in the Society.

While in London, Maxwell attended the lectures at the Royal Institution and had the opportunity for regular contact with Michael Faraday. The ideas and experimental work of Faraday were a

major influence in Maxwell's development of a general electromagnetic theory. Although Faraday was about 40 years older than Maxwell, the two seemed to communicate very well and they greatly admired each others work. In 1861 Maxwell presented a two-part paper entitled *On Physical Lines of Force*. Here he provided a conceptual model for electromagnetics consisting of a network of small spinning cells separated by spherical idle wheels. In 1862 he published two additional parts of this paper. The first dealt with electrostatics, the displacement current produced in a dielectric due to polarization, and the generation of electromagnetic waves. The second dealt with the rotation of the plane of polarization of light in a magnetic field, a phenomenon discovered by Faraday. While this model was able to predict many of the known results in electricity and magnetism, Maxwell realized that this model was purely conceptual and didn't represent actual mechanisms. He didn't pursue this model any further, but began looking for a way to incorporate Faraday's ideas concerning electric and magnetic fields.

Although Maxwell enjoyed his five years in London, the academic demands left little time for the research he loved. Therefore, in 1865, Maxwell resigned the chair at King's College London and returned to Glenlair with his wife Katherine.

While home at Glenlair Maxwell developed the majority of his general theory of electromagnetics, although his *Treatise on Electricity and Magnetism* was not published until 1873. In 1871 he published a textbook on *The Theory of Heat*. In 1871 he also agreed to be the first Cavendish Professor of Physics at Cambridge. In this position he supervised the construction of the world famous Cavendish Laboratory as well as the selection and purchase of the Laboratory equipment. A picture of the Cavendish Laboratory is shown in figure 7. As was mentioned previously, this laboratory over the years has been the home of numerous Nobel prize winners.

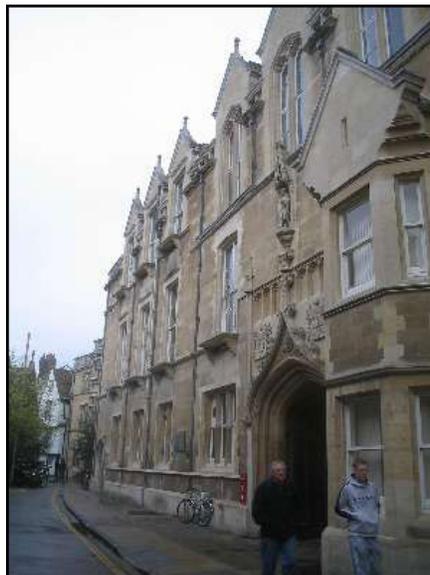


Figure 7: The Cavendish Laboratory

Maxwell died of abdominal cancer in 1879 at the age of 48.

The Faith of James Clerk Maxwell

What we know of Maxwell's faith comes primarily from his notes and letters and from the recollections of his friends. We noted previously that James was raised as both a Presbyterian and an Anglican. His father took him to the Presbyterian church and his aunt took him to the Anglican church. Throughout his life he committed large portions of scripture to memory. At age eight he had memorized the 119th Psalm (176 verses). During Maxwell's time at the University of Cambridge he seems to have subjected his beliefs to serious examination. As a result, his faith grew considerably during this period. In an 1851 letter to his friend Lewis Campbell, he wrote

I believe with the Westminster Divines and their Predecessors ad infinitum that Man's Chief End is to glorify God and to enjoy him for ever.

Maxwell's desire was for all of his beliefs to be open to examination and revision. In another letter to Campbell, dated 7 March 1852, he wrote

Now, my great plan, which was conceived of old, and quickens and kicks periodically, and is continually making itself more obtrusive, is a plan of Search and Recovery, or Revision and Correction, or Inquisition and Execution, etc. The Rule of the Plan is to let nothing be willfully left unexamined. Nothing is to be holy ground consecrated to Stationary Title, whether positive or negative.

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Now I am convinced that no one but a Christian can actually purge his land of these holy spots. Any one may profess that he has none, but something will sooner or later occur to every one to show him that part of his ground is not open to the public. Intrusions on this are resented, and so its existence is demonstrated.

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Christianity—that is, the religion of the Bible—is the only scheme or form of belief which disavows any possessions on such a tenure. Here alone all is free. You may fly to the ends of the world and find no God but the Author of Salvation. You may search the Scriptures and not find a text to stop you in your explorations.

At Trinity college he became friends with Henry and Frank Mackenzie and G.W.H. Tayler who came from a strong evangelical background. In 1853 Maxwell was studying hard for the Mathematics tripos and Smith's Prize examinations. In June he was able to get away from Cambridge for a few days and was invited by G.W.H. Tayler to stay with him at his uncle's house. His uncle was an evangelical rector of Otley, Suffolk. While there Maxwell became seriously ill. The Taylers nursed him back to health over a period of several weeks. Maxwell was very moved by the care and kindness shown him by the Taylers. His biographer, Lewis Campbell, said that "He referred to it long afterwards as having given him a new perception of the Love of God. One of his strongest convictions thenceforward was that 'Love abideth, though Knowledge vanish away.' " After his return to Cambridge, he wrote a letter of thanks to his host that contained the following confession:

All the evil influences that I can trace have been internal and not external, you know what I mean — that I have the capacity of being more wicked than any example that man could set me, and that if I escape, it is only by Gods grace helping me to get rid of myself, partially in science, more completely in society, — but not perfectly except by committing myself to God as the instrument of His will, not doubtfully, but in the certain hope that that Will will be plain enough at the proper time.

We see from this that Maxwell was keenly aware that he was a sinner in need of God's grace.

Maxwell was, by nature, a kind person. One time a fellow student at Cambridge, Charles Robertson, injured his eyes and found it difficult to read. Robertson later wrote

He (Maxwell) used to find me sitting in my room with closed eyes, unable to prepare the next day's lectures, and often gave up an hour of his recreation time to read out to me some of the book-work I wanted to get over.

He frequently cheered up fellow students when they were depressed and often nursed them when they were sick. He also helped freshmen students that were having troubles with their studies. One fellow student, who was not a particular friend, later told Campbell that

Of Maxwell's geniality and kindness of heart you will have many instances. Everyone who knew him at Trinity can recall some kindness or some act of his which left an ineffaceable impression of his goodness on the memory—for "good" Maxwell was in the best sense of the word

While at Cambridge Maxwell became a member of the Conversazione Society, a select group of top students known as the Apostles since they only had twelve members at any one time. This group discussed openly many of the topics of the day, including religion. They believed that "There were no propositions so well established that an Apostle had not the right to deny or question, if he did so sincerely and not from mere love of paradox" Over time this group became very close and many of the members were among Maxwell's closest friends. The discussions with the Apostles served to clarify Maxwell's faith and its relation to science.

Maxwell was also influenced by Frederick Denison Maurice, a former "Apostle" and the founder of the Christian Socialist movement. Although he didn't agree with much of Maurice's theology, he was impressed by Maurice's work with working class men who had no access to higher education. Maurice established a number of Working Men's Colleges where these men were offered college level classes. Maxwell saw this as a vital Christian service and taught weekly classes to working men from 1854 till 1866.

In addition to his studies in mathematics and science, Maxwell loved to read and write poetry. The following are two of his poems that give us a glimpse of his deep faith:

A Student's Evening Hymn

Now no more the slanting rays
With the mountain summits dally,
Now no more in crimson blaze
Evenings fleecy cloudless rally,
Soon shall Night front off the valley
Sweep that bright yet earthly haze,
And the stars most musically
Move in endless rounds of praise.

While the world is growing dim,
And the Sun is slow descending
Past the far horizons rim,
Earth's low sky to heaven extending,
Let my feeble earth-notes, blending
With the songs of cherubim,
Through the same expanse ascending,
Thus renew my evening hymn.

Thou that fillst our waiting eyes
With the food of contemplation,
Setting in thy darkened skies
Signs of infinite creation,
Grant to nightly meditation
What the toilsome day denies —
Teach me in this earthly station
Heavenly Truth to realise.

Give me wisdom so to use
These brief hours of thoughtful leisure,
That I may no instant lose
In mere meditative pleasure,
But with strictest justice measure
All the ends my life pursues,
Lies to crush and truths to treasure,
Wrong to shun and Right to choose.

Then, when unexpected Sleep,
O'er my long-closed eyelids stealing,
Opens up that lower deep
Where Existence has no feeling,
May sweet Calm, my languor healing,

Lend me strength at dawn to reap
All that Shadows, world-concealing,
For the bold enquirer keep.

Through the creatures Thou hast made
Show the brightness of Thy glory,
Be eternal Truth displayed
In their substance transitory,
Till green Earth and Ocean hoary,
Massy rock and tender blade
Tell the same unending story —
“We are Truth in Form arrayed.”

When to study I retire,
And from books of ancient sages
Glean fresh sparks of buried fire
Lurking in their ample pages —
While the task my mind engages
Let old words new truths inspire —
Truths that to all after-ages
Prompt the Thoughts that never tire.

Yet if, led by shadows fair
I have uttered words of folly,
Let the kind absorbing air
Stifle every sound unholy.
So when Saints with Angels lowly
Join in heavens unceasing prayer,
Mine as certainly, though slowly,
May ascend and mingle there.

Teach me so Thy works to read
That my faith, — new strength accruing, —
May from world to world proceed,
Wisdom’s fruitful search pursuing;
Till, thy truth my mind imbuing,
I proclaim the Eternal Creed,
Oft the glorious theme renewing
God our Lord is God indeed.

Give me love aright to trace
Thine to everything created,
Preaching to a ransomed race

By Thy mercy renovated,
Till with all thy fulness sated
I behold thee face to face
And with Ardour unabated
Sing the glories of thy grace.

To My Wife

Oft in the night, from this lone room
I long to fly oer land and sea,
To pierce the dark, dividing gloom,
And join myself to thee.

And thou to me wouldst gladly fly,
I know thee well, my own true wife!
We feel, that when we live not nigh,
We lose the crown of life.

Yet soon I hope, at dead of night,
To meet where all is strange beside,
And mid the trains resounding flight
To have thee by my side.

Then shall I feel that thou art near,
Joined hand to hand and soul to soul;
Short will that happy night appear,
As through the dark we roll.

Then shall the secret of the will,
That dares not enter into bliss;
That longs for love, yet lingers still,
Be solved in one long kiss.

I, drinking deep of thy rich love,
Thou feeling all the strength of mine,
Our souls will rise in faith above
The cares which make us pine.

Till I give thee, thou giving me,
As that which either loves the best,
To Him that loved us both, that He

May take us to His rest.

Wandering and weak are all our prayers,
And fleeting half the gifts we crave;
Love only, cleansed from sins and cares,
Shall live beyond the grave.

Strengthen our love, O Lord, that we
May in Thine own great love believe
And, opening all our soul to Thee,
May Thy free gift receive.

All powers of mind, all force of will,
May lie in dust when we are dead,
But love is ours, and shall be still,
When earth and seas are fled.

Maxwell and his wife Katherine shared a common Christian faith. We have a number of letters that Maxwell wrote to his wife Katherine both before and after their marriage. In these letters they often discussed the meaning of various passages of scripture. Here is one example

Now let us read (2 Cor.) chapter xii., about the organisation of the Church, and the different gifts of different Christians, and the reason of these differences that Christ's body may be more complete in all its parts. If we felt more distinctly our union to Christ, we would know our position as members of His body, and work more willingly and intelligently along with all the rest in promoting the health and growth of the body, by the use of every power which the spirit has distributed to us.

Throughout their marriage they read the scriptures together nightly. When Maxwell returned to London to teach at King's College, they often worshiped with a Baptist congregation. In a letter to Rev. Tayler he wrote

At Cambridge I heard several sermons from excellent texts, but all either on other subjects or else right against the text. There is a Mr. Offord in this street, a Baptist who knows his Bible, and preaches as near it as he can, and does what he can to let the statements in the Bible be understood by his hearers. We generally go to him when in London, though we believe ourselves baptized already.

In 1864, while at King's College, Maxwell wrote

Think what God has determined to do to all those who submit themselves to his righteousness and are willing to receive his gift [the gift of eternal life in Jesus Christ]. They are to be conformed to the image of His Son, and when that is fulfilled, and God sees they are conformed to the image of Christ, there can be no more condemnation.

During Maxwell's lifetime many evangelicals were becoming concerned with the increasing influence of Darwinism and scientific naturalism outside the church and with biblical criticism within the church. To combat these trends the Victoria Institute was formed in 1865. It consisted of evangelical clergy, lay people, and a few university professors. Maxwell was frequently asked to join this society. Although he rejected Darwinism, he politely refused their invitation. One reason was that he didn't like the militant tone of the society and the way it attacked some of his friends and fellow scientists such as Adam Sedgwick and William Thomson (Lord Kelvin). He was also very hesitant to tie current formulations of science with biblical interpretation. Below are a couple of his statements on this subject

The rate of change of scientific hypothesis is naturally so much more rapid than that of biblical interpretations, so that if an interpretation is founded on such an hypothesis, it may help to keep the hypothesis above ground long after it ought to be buried and forgotten.

But I think that the results which each man arrives at in his attempts to harmonize his science with his Christianity ought not to be regarded as having any significance except to the man himself and to him only for a time and should not receive the stamp of a society.

When Maxwell took over the running of the Glenlair estate after his father's death, he led a daily prayer service for his servants and staff. We don't have any record of the actual prayers used, but the following two prayers were found in his notes:

Almighty God, who hast created man in Thine own image, and made him a living soul that he might seek after Thee and have dominion over Thy creatures, teach us to study the works of Thy hands that we may subdue the earth to our use, and strengthen our reason for Thy service; and so to receive Thy blessed Word, that we may believe on Him whom Thou hast sent to give us the knowledge of salvation and the remission of our sins. All which we ask in the name of the same Jesus Christ our Lord.

O Lord, our Lord, how excellent is Thy name in all the earth, who hast set Thy glory above the heavens, and out of the mouths of babes and sucklings hast perfected praise. When we consider Thy heavens, the work of Thy fingers, the moon and the stars which Thou hast ordained, teach us to know that Thou art mindful of us, and visitest us, making us rulers over the works of Thy hands, showing us the wisdom of Thy laws, and crowning us with honour and glory in our earthly life; and looking higher than

the heavens, may we see Jesus, made a little lower than the angels for the suffering of death, crowned with glory and honour, that He, by the grace of God, should taste death for every man. O Lord, fulfil Thy promise, and put all things in subjection under His feet. Let sin be rooted out of the earth, and let the wicked be no more. Bless Thou the Lord, O my soul, praise the Lord.

At 48, in 1879, when his abdominal cancer was getting extremely painful, the minister who visited him was amazed by Maxwell's clarity and memory. He said that Maxwell never complained and his kindness did not subside. Moreover,

... his illness drew out the whole heart and soul and spirit of the man: his firm and undoubting faith in the Incarnation and all its results; in the full sufficiency of the Atonement; in the work of the Holy Spirit. He had gauged and fathomed all the schemes and systems of philosophy, and had found them utterly empty and unsatisfying — “unworkable” was his own word about them — and he turned with simple faith to the Gospel of the Savior.

During this time he confessed to the Reverend Professor Holt that “what is done by what I call myself is, I feel, done by something greater than myself in me.” Near the end, Maxwell told a colleague, “The only desire which I can have is like David to serve my own generation by the will of God, and then fall asleep.” His burial place in Galloway Scotland is shown in figure 8.



Figure 8: Maxwell's burial site in Southern Scotland

Maxwell's Contributions to Science

In addition to his revolutionary work in electromagnetism, Maxwell made contributions to many areas of science. These include geometry, color perception, astronomy, statistical mechanics, thermodynamics, elasticity, control theory, and dimensional analysis. In this section we will look at some of these.

Geometry

Maxwell's first contributions to science were in mathematics, in particular geometry. His first paper was entitled *On the description of Oval Curves, and those having a plurality of foci* and was published in the Proceedings of the Royal Edinburgh Society. Maxwell was fourteen at the time. In this paper he discussed oval curves that could be drawn using pins, a piece of string and a pencil. It was well known that an ellipse consists of those points such that the sum of the distances from each point to two fixed points (foci) is a constant. An ellipse can be drawn as shown in figure 9.

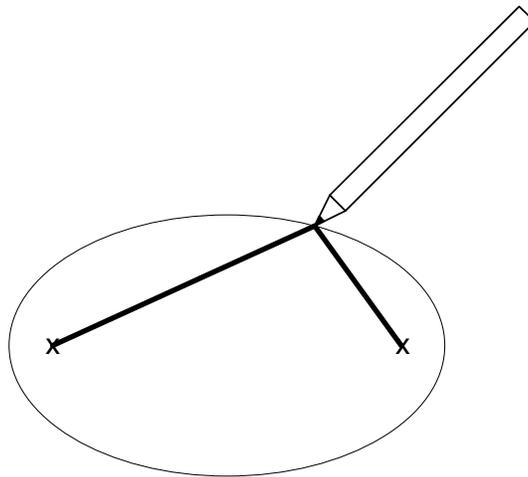


Figure 9: Drawing an ellipse

Here a piece of string is pinned at its two ends, the pencil is pushed against the string and moved keeping the string taut. Maxwell generalized this construction method to other types of ovals. For example, if you unpin one of the ends, tie this end of the string to the pencil, loop the string around the unused pin, press the pencil against the loop of string, and draw, you get an oval in which the sum of twice the distance to one of the pins plus the distance to the other pin is a constant. Using multiple loops he obtained many additional ovals. He also considered ovals constructed using three, four and five pins. Maxwell was deemed too young to present the paper to the society, so the paper was read by professor Forbes of the University of Edinburgh. It was discovered after this paper was published that many of the ovals Maxwell generated had been discovered previously by the great French mathematician Descartes. However, Maxwell's paper was more general and his construction methods simpler.

Maxwell's second paper was presented to the Royal Edinburgh Society when he was eighteen. It was entitled *Rolling Curves*. A simple example of a rolling curve is the cycloid generated by a fixed point on a circle as it rolls along on a line (see figure 10). Maxwell considered general properties of curves generated by a point on one curve as it rolls on a second curve.



Figure 10: Cycloid generated by a point on a circle that is rolling on a line

Color Theory

Today James Clerk Maxwell is best known for his electromagnetic equations, but during his lifetime he was better known for his work on color perception. Newton was the first to develop a color theory. He separated colors into a rainbow-like spectrum using a prism, and showed that white was the combination of all the spectral colors. He also postulated that any color could be obtained by mixing the spectral colors in the correct proportions. However, artists commonly obtained the colors they desired by mixing just three colors of paint (usually red, yellow, and blue).

In the early nineteenth century, an English doctor and physicist Thomas Young postulated that the human eye contains three receptors each sensitive to a particular color. He also suggested the idea of representing colors in a triangle with the primary colors at the vertices, however, he didn't follow up on this idea. Maxwell began his work on color in Professor Forbes laboratory while he was a student at Edinburgh University (1848–1850). They were trying to show that any color could be obtained as a mixture of three primary colors. Various colors were produced by spinning a circular disk having colored sectors. If the disk is spun rapidly enough, the eye can not separate the colors and the disk appears to be of a uniform color. They tried to obtain white by using sectors colored red, yellow, and blue (the colors used by painters), but no combination of these three colors produced white. They also used yellow and blue sectors in an attempt to obtain green. To their surprise, the resulting color was a dull pink.

James soon found out the reason for these discrepancies. There is a fundamental difference between mixing colored lights and mixing paint pigments. Paint pigments tend to absorb certain colors, so that the color you see consists of the colors not absorbed. Thus, mixing paint pigments is a subtractive process. However, mixing colored lights is an additive process. When James went to red, green, and blue sectors he was able to produce white and a variety of other colors. He also showed that any three colors could be used as primaries as long as white could be obtained as some combination of these colors.

Maxwell later improved on the spinning disk by adding a smaller sectored disk in the center. He

could then easily compare the color of the outer portion of the disk with the inner portion. This device is usually called Maxwell's top and is pictured in figure 11. Maxwell could vary intensity as well as color by adding black sectors to the inner or outer disk. For example, various intensities of white (shades of gray) could be obtained in the center by using black and white sectors as shown in figure 11.

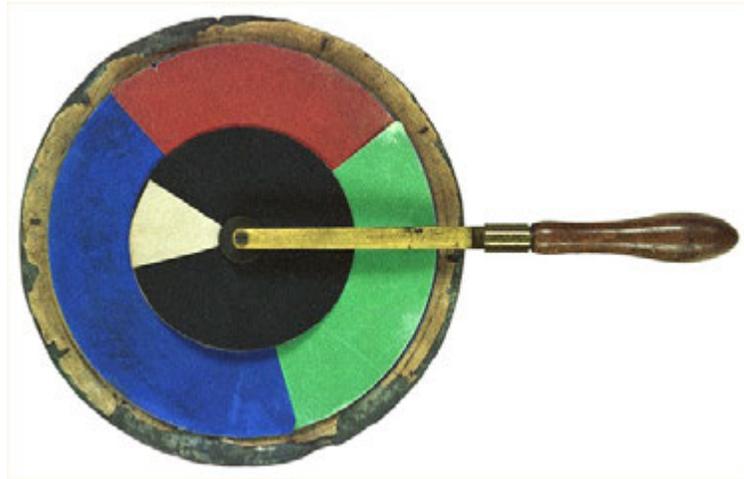


Figure 11: Maxwell's top

Maxwell realized that color was not purely a physical property of an observed object, but that it also depended on how the light was processed by the eye. He agreed with Young that the eye must contain three type of color sensors. However, he believed that while each of these sensors was most sensitive to one color, they were also sensitive to a lesser degree to nearby colors. Maxwell also was aware that certain colors could only be matched by having a negative amount of at least one primary. He achieved this with his top by combining the color to be matched with one of the primaries on either the inner or outer disk and then matching a combination of the other two primaries on the other portion of the disk.

As useful as the color top was, Maxwell realized that it was limited by the relatively few colors of paper that were available. He designed several color boxes that produced spectral colors using a combination of prisms. The prisms spread out the spectral colors spatially and the desired colors and their amounts were controlled by precisely calibrated adjustable slits. The colors were combined by means of a series of mirrors and lenses. He was able to obtain much more accurate measurements using these boxes. He was also able to associate the colors mixed with their wavelengths. Taking measurements on a large number of subjects, he was able to determine the cause of common types of color-blindness. Those who were color blind were deficient in one of the three color receptors, usually the red receptor. Maxwell even invented one of the first optical instruments to view the retina. Maxwell is also credited with producing the first color photograph. He took photographs using red, green, and blue filters. He the combined them by projecting the photographs using the same filters. However, Maxwell's biggest contribution to color perception was his development of a mathematical model for describing colors and their combinations.

The most complete description of his model was contained in the article [James Clerk Maxwell, *On the Theory of Compound Colours, and the Relations of the Colours of the Spectrum*, Phil. Trans. R. Soc. Lond., **150**, pp 57–84, 1860]. Maxwell represented the three primary colors by three points in a plane that are the vertices of a triangle. He selected an origin that doesn't lie in the plane of the triangle. Arbitrary colors are represented by points in three-space. Each point has an associated vector from the origin to the point. In Maxwell's model colors are combined in the same way that force vectors are combined in mechanics. Thus, the vector associated with any color can be represented by a linear combination of the vectors associated with the three primaries. The vector associated with a color has both a direction and a magnitude. The direction is determined by the point where an extension of the vector intersects the plane of the color triangle. This point of intersection he called the quality of the color. The ratio of the length of the vector associated with a color to the length of the vector associated with the associated quality point is called the quantity of the color. The quality is a measure of the shade or tint of a color and the quantity is a measure of its intensity. All the points in the plane of the triangle have a quantity of one. A more detailed description of Maxwell's color model is contained in Appendix A. Figure 12 shows the colors in a Maxwell triangle (It doesn't show the colors that lie outside the triangle).

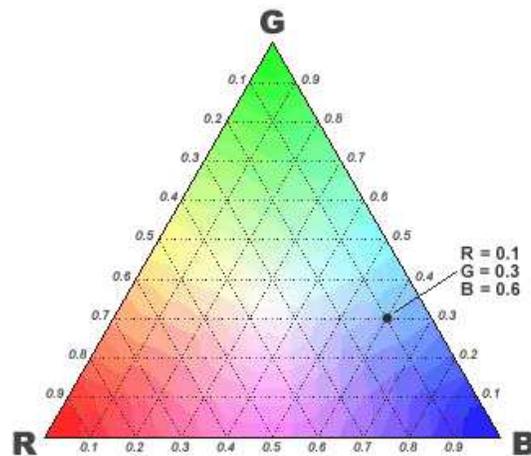


Figure 12: A Maxwell color triangle

Maxwell received the Rumford medal from the Royal Society of London in 1860 for his work on color vision.

Saturn's Rings

Saturn, with its large collection of flat rings, had been a mystery to astronomers for centuries. How could such a strange arrangement be stable? Why didn't the rings collapse into Saturn or drift off into space? In 1855 St. John's College of Cambridge posed this topic for their Adams' prize. They asked under what conditions the rings would be stable if they were a solid, a fluid, or were made up of independently orbiting pieces of matter. Responses were due by December 1857. Maxwell worked a whole year on this problem and was the only participant to enter a solution.

Using very complicated mathematical calculations, he was able to show that solid rings would be unstable except for one unreasonable situation in which four-fifths of the mass was concentrated on one point of the circumference and the rest was evenly distributed. For the fluid case Maxwell used Fourier analysis to study the various types of waves that could exist. He showed that the fluid rings would eventually break up into separate blobs. By a process of elimination he had shown that the only possible configuration was for the rings to be made up of individually orbiting pieces of matter. He wasn't able to establish stability for this case in general, but he did analyze a special case in which there was a single ring made up of equally spaced particles. He showed that the ring would be stable if its average density was small enough compared with that of Saturn. James was awarded the Adams prize in 1859 and his conclusion about the make-up of the rings was verified by Voyager flybys in the 1980s. Maxwell's essay was so detailed and convincing that the English mathematician and astronomer George Airy wrote

It is one of the most remarkable applications of mathematics to physics that I have ever seen.

The Kinetic Theory of Gases

If Maxwell had made no other contributions to science, his work on the kinetic theory of gases would have established him as one of the greatest physicists of all time. He was the first to introduce statistical laws into physics. This has led to the use of statistics in fields such as statistical mechanics, thermodynamics, and quantum mechanics. The kinetic theory of gases originated with the work of the eighteenth century mathematician Daniel Bernoulli. He proposed that gases consist of a large number of particles moving in all directions. In addition, he assumed that the pressure exerted by a gas on a surface is due to the impact of these particles and that heat is merely the kinetic energy of these particles. This theory was developed by others, and many of the properties of gases could be explained on this basis. But there was one major difficulty that had yet to be explained. In order for the theory to describe the pressures generated at normal temperatures, the particles must move very rapidly. Why then do gases diffuse so slowly? For example, the spread of odors throughout a room is a slow process. In 1859 Maxwell read a paper by the German physicist Rudolf Clausius who proposed that the slow rate of diffusion was due to a large number of collisions between the particles. This paper stimulated Maxwell's interest in the subject. Maxwell realized that it was impossible to describe all the motions and collisions exactly using standard Newtonian mechanics. He therefore derived a statistical law for the distribution of velocities. He represented each particle velocity in terms of its three components relative to an arbitrary set of orthogonal axes. The speed of the particle would then be the square root of the sum of the squares of the three components. He assumed that the three components of velocity were statistically independent and that each component had the same probability distribution, which he took as Gaussian or normal. The distribution $f(v)$ of particle speeds then has the form

$$f(v) = Av^2e^{-\alpha v^2}.$$

This distribution is known as the Maxwell distribution or the Maxwell-Boltzmann distribution of

velocities. Maxwell used his formulation of the kinetic theory to derive an unexpected result, namely that the viscosity of a gas is independent of both pressure and density. The same was true of thermal conductivity. These results he later confirmed experimentally. Maxwell's work inspired a young Austrian physicist named Ludwig Boltzmann and he, along with Maxwell, further developed the kinetic theory of gases.

Maxwell's Demon

Maxwell's demon is the name given by Maxwell's friend William Thomson (Lord Kelvin) to a hypothetical figure that appeared in a letter Maxwell sent to Peter Tait upon receipt of a manuscript of Tait's book **Thermodynamics** for review. This hypothetical creature was the main player in a thought experiment involving the second law of thermodynamics (one of the consequences of this law is that heat can not be transferred from a cold to a hot body without the expenditure of work). Here is this thought experiment in Maxwell's own words

Now let A & B be two vessels divided by a diaphragm and let them contain elastic molecules in a state of agitation which strike each other and the sides. Let the number of particles be equal in A & B but let those in A have the greatest energy of motion I have shown that there will be velocities of all magnitudes in A and the same in B only the sum of the square of the velocities is greater in A than in B. When a molecule is . . . allowed to go through a hole in the [diaphragm] . . . no work would be lost or gained only its energy would be transferred from the one vessel to the other.

Now conceive a finite being who knows the paths and velocities of all the molecules by simple inspection but who can do no work, except to open and close a hole in the diaphragm, by means of a slide without mass. Let him first observe the molecules in A and when he sees one coming the square of whose velocity is less than the mean sq. vel. of the molecules in B let him open the hole & let it go into B. Next let him watch for a molecule in B the square of whose velocity is greater than the mean sq. vel. in A and when it comes to the hole let him draw the slide & let it go into A, keeping the slide shut for all other molecules.

Then the number of molecules in A & B are the same as at first but the energy in A is increased and that in B diminished that is the hot system has got hotter and the cold colder & yet no work has been done, only the intelligence of a very observant and neat fingered being has been employed.

This was one of the first examples of a thought experiment in physics. Maxwell's intent was not to disprove the second law, but to show that this law had only statistical certainty and could not be applied to individual molecules. In Maxwell's review of the second edition of Tait's **Thermodynamics** in 1878 he notes that *the second law of thermodynamics is continually being violated, and that to a considerable extent, in any sufficiently small group of molecules belonging to a real body.*

Maxwell's Electromagnetic Equations

Maxwell's greatest scientific accomplishment was certainly his development of a general theory for electromagnetism and the use of this theory to predict the existence of electromagnetic waves. This achievement places him alongside Newton and Einstein as one of the greatest physicists of all time. There had been a number of important developments involving electricity and magnetism leading up to Maxwell's discovery.

In 1784 Charles-Augustin de Coulomb discovered that the force of attraction or repulsion between two small electrified bodies varies inversely as the square of the distance between them. In 1799 Alessandro Volta developed a zinc and silver in brine battery that provided a source for continuous electric currents. In 1820 Hans-Christian Oersted observed that a current in a wire caused a nearby compass needle to deflect. Less than a week after hearing of Oersted's discovery, André-Marie Ampère presented a paper before the French Academy of Sciences giving a more complete treatment of this phenomena and demonstrated that parallel current carrying wires attract each other if the currents are in the same direction and repel each other if the currents are in opposite directions. The following year Michael Faraday produced a primitive electric motor based on these results.

In 1827 Ampère showed that two circular current carrying coils behave as magnets with the force of attraction or repulsion varying inversely as the square of the distance between them. In 1831 Faraday discovered that a changing magnetic flux through a wire loop will produce an electric current in the wire. Faraday used this result to construct an electrical generator. During Maxwell's lifetime there were a number of scientists attempting to formulate a general theory for electricity and magnetism. Since the forces between electrical charges or magnetic poles obey inverse-square laws similar to Newton's law for the gravitational attraction between masses, most investigators approached the task using this instantaneous action at a distance force model. It is interesting that Newton himself was troubled by the fact that distant objects could interact instantaneously without any evident connection between them. He once said

That gravity should [be such] that one body can act upon another at a distance through a vacuum, without the mediation of anything else . . . is to me so great an absurdity, that I believe no man [of] competent faculty of thinking can ever fall into it.

However, he was so successful in predicting the motions of planets and other objects using this approach that scientists soon forgot about the philosophical difficulties. Ampère and Weber were probably the two most influential scientists at the time of Maxwell that were committed to the Newtonian action at a distance approach to electricity and magnetism. Michael Faraday, however, had a different idea. He couldn't believe that forces appeared instantaneously at a distance. He thought that something must be happening in between and that forces must be transmitted between magnetic poles or electrical charges. He was very impressed with the patterns formed by iron filings sprinkled on a paper with a magnet underneath (see figure 13). He called the lines formed by these particles "lines of force".

Faraday introduced the idea of electric and magnetic fields acting throughout space. He felt that forces were transmitted through the interaction of these fields. Since Faraday had no mathemat-

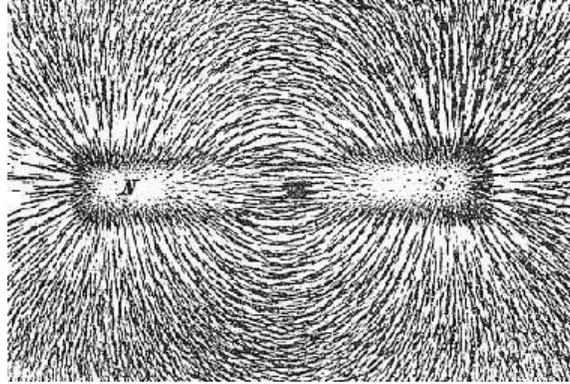


Figure 13: Patterns of iron filings produced by a magnet.

ical training, his ideas were not embraced by the scientific community. Maxwell, however, was intrigued by these ideas and attempted to give them a mathematical basis. His first paper dealing with this topic was called *On Faraday's Lines of Force*. He sent a copy to Faraday. Faraday was delighted to have someone interested in his ideas and sent the following reply

I received your paper, and thank you very much for it. I do not say I venture to thank you for what you have said about 'Lines of Force', because I know you have done it for the interests of philosophical truth; but you must suppose it is work grateful to me, and it gives me much encouragement to think on. I was almost frightened when I saw such mathematical force made to bear upon the subject, and then wondered that the subject stood it so well. I send by this post another paper to you; I wonder what you will say to it . . .

Maxwell had numerous conversations with Faraday while he was in London, and he developed a great respect for this aging scientist. Faraday was flattered that a young scientist of Maxwell's caliber was interested in his ideas.

In 1861 Maxwell presented a two part paper entitled [*On Physical Lines of Force*, *Philosophical Magazine and Journal of Science* (March 1861)]. Two more parts of this paper were added in 1862. In this paper he developed a conceptual elastic model involving a network of tiny spinning cells separated by spherical idle wheels. With this model he was able to duplicate all the known behaviors of electricity and magnetism. However, Maxwell knew that this model was merely an analogy and didn't represent actual mechanisms. In fact, Maxwell seemed to sense that the actual mechanisms may be beyond our ability to grasp.

Instead of refining this elastic model, he turned to more of a black-box approach in which the emphasis was more on describing the mathematical relationships between the resulting forces (fields) than on the underlying mechanisms. He was eventually able to develop a model based on electric and magnetic fields that we now refer to as Maxwell's Equations. In deriving these equations Maxwell relied heavily on the emerging field of vector analysis. In fact, Maxwell coined the term "curl" for one of the common vector differential operators. Maxwell originally stated his results

in eight equations involving electric and magnetic potentials, but Oliver Heaviside later condensed them to the familiar four equations shown below.

$$\operatorname{div} \mathbf{E} = 4\pi\rho \quad (1a)$$

$$\operatorname{div} \mathbf{B} = 0 \quad (1b)$$

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (1c)$$

$$\operatorname{curl} \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad (1d)$$

Here \mathbf{E} is the electric field vector, \mathbf{B} is the magnetic induction vector, ρ is the free charge density, and \mathbf{J} is the current density. The term “div” stands for “divergence” and represents a differential operator that measures the tendency, on average, of a vector field to point inward or outward from a point. If the divergence is positive, there is an outward tendency indicating the presence of a source at the point. If the divergence is negative, there is an inward tendency indicating a sink at the point. When the divergence is zero, it indicates that there is neither a source nor a sink at the point. The term “curl” represents a differential operator that measures the tendency of a vector field to wrap around a point. The sign indicates whether it wraps in a clockwise or counter-clockwise direction.

Equation (1a) is called Gauss’ law and is a generalization of Coulomb’s inverse square law for point charges. Equation (1b) states that there are no magnetic point sources or sinks. Magnetic lines of force never originate or terminate at a point, but always form closed loops. Equation (1c) is a statement of Faraday’s law of magnetic induction. Equation (1d) is a statement of Ampere’s law for magnetic-like forces generated by currents with an added term that is Maxwell’s major contribution. The added second term in equation (1d) is called the displacement current and represents the current generated by a changing electric field analogous to the way that a changing magnetic field generates a current. Maxwell showed that the constant c appearing in these equations was very close to the measured velocity of light.

Maxwell used these equations to derive wave equations for both \mathbf{E} and \mathbf{B} (see Appendix B). Because of these results Maxwell predicted the existence of electromagnetic waves and postulated that light was such a wave. It should be noted that electromagnetic waves involve both the electric field and the magnetic induction field. There are no purely electrical or purely magnetic waves. Moreover, all electromagnetic waves travel at the speed of light.

The extra displacement current term added by Maxwell was crucial to the derivation of these results. At the time displacement currents had not been measured, but Maxwell felt that they must exist. The American physicist Henry Rowland made a direct measurement of this added term in 1875. It was not until 1887, eight years after Maxwell’s death, that Heinrich Hertz produced and measured an electromagnetic wave other than light.

Maxwell’s equations play a similar role in electromagnetics as Newton’s laws play in mechanics. It is interesting that Newton’s laws had to be modified in Einstein’s theory of relativity, but Maxwell’s equations remained unchanged. In fact, Maxwell’s equations were one of the main motivations for

Einstein's theory of relativity. The concept of fields introduced by Faraday and refined by Maxwell now plays an important part in all branches of physics. In 1873 Maxwell published the two volumes of his famous work *A Treatise on Electricity and Magnetism*. It is difficult to over state the effect that Maxwell's equations have had on the development of modern physics and engineering.

Other Contributions

Maxwell published a famous paper On governors in the Proceedings of Royal Society, vol. 16 (1867-1868). This paper is quite frequently considered a classical paper of the early days of control theory. Here governors refer to the governor or the centrifugal governor used in steam engines.

Maxwell was also asked by the British Association for the Advancement of Science to lead a small team whose goal would be to sort out the hodgepodge of units being used in electricity and magnetism. Maxwell realized that the confusion over units was not confined to electricity and magnetism. He proposed a systematic way of defining all physical quantities in terms of mass, length and time (**M**, **L**, and **T**), e.g., velocity has dimension L/T , acceleration has dimension L/T^2 , and force has dimension ML/T^2 . His work in this regard became the basis for what is now called dimensional analysis. Dimensional analysis is often used to verify that all terms in an equation have the same dimensions. The committee presented their report in 1863. The report contained a standard system of units that was later adopted, almost unchanged, as the first international set of units (misleadingly called Gaussian units). The report also looked at ways of measuring electrical and magnetic quantities that only involved mass, length, and time.

Appendix A: Maxwell's color model

As was mentioned in the main article, Maxwell's color model was described in the paper [James Clerk Maxwell, *On the Theory of Compound Colours, and the Relations of the Colours of the Spectrum*, Phil. Trans. R. Soc. Lond., **150**, pp 57–84, 1860]. Maxwell represented the three primary colors by three points in a plane that are the vertices of a triangle. Let us denote these points by **R**, **G**, and **B**. He selected an origin **O** that doesn't lie in the plane of the triangle. Arbitrary colors are represented by points in three-space. Each point has an associated vector from the origin to the point. We will use the same symbol to represent a point in three-space and its associated vector. Let **C** represent some color. The vector associated with **C** has a length and a direction. The direction is uniquely determined by the point of intersection of the line through **O** and **C** with the plane determined by the points **R**, **G**, and **B**. Maxwell called this point in the **RGB**-plane the quality of the color. The length of the vector relative to the length of the corresponding quality point vector he called the quantity (intensity) of the color. Thus, every point in the **RGB**-plane has quantity one. The vector associated with any color can be uniquely represented by a linear combination of the linearly independent vectors associated with **R**, **G**, and **B**. Let us now look at how Maxwell represented mixes of colors. In Maxwell's model colors are combined in the same way as force vectors are combined in mechanics. Let **P** and **Q** be two points in the plane of **R**, **G**, and **B** (see figure 14).

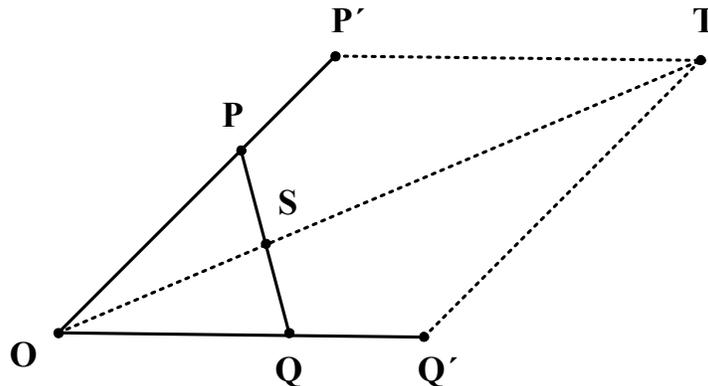


Figure 14: Combination of colors

Suppose we want to mix p units of **P** and q units of **Q**. Let **P'** and **Q'** be the points (vectors) given by

$$\mathbf{P}' = p\mathbf{P} \quad \text{and} \quad \mathbf{Q}' = q\mathbf{Q}. \quad (2)$$

These points will generally not lie in the plane of the triangle. Let **T** be the vector sum of **P'** and **Q'**. Define **S** to be the point of intersection of the line **OT** and the line **PQ**, i.e., **S** is the quality of the vector sum **T**. Then,

$$\mathbf{T} = \mathbf{P}' + \mathbf{Q}' = p\mathbf{P} + q\mathbf{Q} = \gamma\mathbf{S} \quad \text{for some scalar } \gamma. \quad (3)$$

Thus, γ is the quality of \mathbf{T} . Since \mathbf{S} lies on the line segment joining \mathbf{P} and \mathbf{Q} , we have

$$\mathbf{S} = \alpha\mathbf{P} + \beta\mathbf{Q} \quad \text{where } \alpha + \beta = 1. \quad (4)$$

Therefore,

$$\mathbf{T} = \gamma\mathbf{S} = \gamma\alpha\mathbf{P} + \gamma\beta\mathbf{Q} = p\mathbf{P} + q\mathbf{Q}. \quad (5)$$

Since every point in the plane determined by \mathbf{O} , \mathbf{P} , and \mathbf{Q} has a unique representation as a linear combination of \mathbf{P} and \mathbf{Q} it follows from equation (5) that

$$\gamma\alpha = p \quad \text{and} \quad \gamma\beta = q. \quad (6)$$

Dividing these two relations we obtain

$$\frac{\alpha}{\beta} = \frac{p}{q} \quad \text{or} \quad \alpha = \frac{p}{q}\beta. \quad (7)$$

Combining equation (7) with the relation $\alpha + \beta = 1$, we get

$$\beta = \frac{q}{p+q} \quad \text{and} \quad \alpha = \frac{p}{p+q}. \quad (8)$$

Since $\gamma\alpha = p$, it follows that $\gamma = p + q$. Thus,

$$\text{quality of } (p\mathbf{P} + q\mathbf{Q}) = \mathbf{S} = \frac{p}{p+q}\mathbf{P} + \frac{q}{p+q}\mathbf{Q} \quad (9)$$

and

$$\text{quantity of } (p\mathbf{P} + q\mathbf{Q}) = \gamma = p + q. \quad (10)$$

These are the basic mixing rules for colors.

We have mentioned that any color can be represented by a linear combination of \mathbf{R} , \mathbf{G} , and \mathbf{B} . In particular, any point in the plane of \mathbf{R} , \mathbf{G} , and \mathbf{B} can be represented by a linear combination of the three primaries in which the sum of the multiplier coefficients is one, i.e., any color \mathbf{C} in the plane of the triangle has a representation of the form

$$\mathbf{C} = r\mathbf{R} + g\mathbf{G} + b\mathbf{B} \quad \text{with} \quad r + b + g = 1. \quad (11)$$

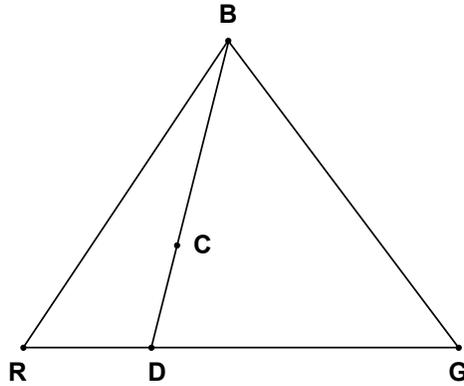


Figure 15: Representation of colors in the RGB-plane

To see this consider a color \mathbf{C} that is inside the triangle as shown in figure 15.

Since \mathbf{C} is on the line joining \mathbf{B} and \mathbf{D} , we have

$$\mathbf{C} = \alpha\mathbf{B} + \beta\mathbf{D} \quad \text{where } \alpha + \beta = 1. \quad (12)$$

Since \mathbf{D} lies on the line joining \mathbf{R} and \mathbf{G} , we have

$$\mathbf{D} = \gamma\mathbf{R} + \delta\mathbf{G} \quad \text{where } \gamma + \delta = 1. \quad (13)$$

combining equations (12) and (13), we get

$$\mathbf{C} = \beta\gamma\mathbf{R} + \beta\delta\mathbf{G} + \alpha\mathbf{B}. \quad (14)$$

Moreover, the sum of the coefficients in equation (14) satisfies

$$\beta\gamma + \beta\delta + \alpha = \beta(\gamma + \delta) + \alpha = \beta + \alpha = 1. \quad (15)$$

A similar argument can be made for colors outside the triangle. If \mathbf{C} is inside the triangle, the coefficients are all positive. For those colors that lie outside the triangle, at least one of the coefficients is negative.

Maxwell also showed that there was a geometric relation between his method of representing colors and another suggested by Hermann Grassmann. Any point in the Plane determined by \mathbf{R} , \mathbf{G} , and \mathbf{B} can be described in terms of a vector from the point representing white in the triangle to the point in question. The direction of this vector in the plane is related to the hue and the magnitude of this vector is related to the saturation.

Appendix B: Derivation of Electromagnetic Wave Equations

In a vacuum Maxwell's equations reduce to

$$\operatorname{div} \mathbf{E} = 0 \quad (16)$$

$$\operatorname{div} \mathbf{B} = 0 \quad (17)$$

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (18)$$

$$\operatorname{curl} \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}. \quad (19)$$

Any vector field \mathbf{F} satisfies the identity

$$\operatorname{curl} \operatorname{curl} \mathbf{F} = \nabla \operatorname{div} \mathbf{F} - \Delta \mathbf{F} \quad (20)$$

where ∇ is the gradient operator and Δ is the Laplacian. Applying equation (20) to \mathbf{E} and \mathbf{B} and making use of equations (16) and (17), we get

$$\operatorname{curl} \operatorname{curl} \mathbf{E} = -\Delta \mathbf{E} \quad (21)$$

$$\operatorname{curl} \operatorname{curl} \mathbf{B} = -\Delta \mathbf{B}. \quad (22)$$

Taking the curl of equation (18) and applying (21), we get

$$\Delta \mathbf{E} = \frac{1}{c} \frac{\partial \operatorname{curl} \mathbf{B}}{\partial t}. \quad (23)$$

Substituting equation (19) into equation (23), we obtain the wave equation

$$\Delta \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}. \quad (24)$$

In a similar manner we can show that \mathbf{B} satisfies the wave equation

$$\Delta \mathbf{B} = \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2}. \quad (25)$$

It is well known that differential equations such as (24) and (25) possess wave-like solutions.

To see that there can be no purely electrical waves, suppose that $\mathbf{B} = \mathbf{0}$. Then it follows from equation (19) that the time derivative of \mathbf{E} is zero, i.e., \mathbf{E} doesn't change with time. In a similar way we can see that there can be no purely magnetic waves.

References

- [1] Mahon, Basil, **The Man Who Changed Everything: The Life of James Clerk Maxwell**, Wiley (2003).
- [2] Harman, P.M., **The Natural Philosophy of James Clerk Maxwell**, Cambridge University Press (1998).
- [3] Campbell, Lewis and Garnett, William, **The Life of James Clerk Maxwell**, London, Macmillan and Co. (1882), available on the internet at <http://www.sonnetsoftware.com/resources/maxwell-bio.html>.
- [4] Seitz, Frederick, *James Clerk Maxwell (1831–1879); Member APS 1875*, Proceedings of the American Philosophical Society, vol. 145, No. 1 (March 2001).
- [5] Maxwell, J. Clerk, *On the Theory of Compound Colours, and the Relations of the Colours of the Spectrum*, Phil. Trans. R. Soc. Lond., **150**, 57–84 (1860).
- [6] Maxwell, J. Clerk, *On Physical Lines of Force*, Philosophical Magazine and Journal of Science (March 1861), available on the internet at http://www.vacuum-physics.com/Maxwell/maxwell_oplf.pdf.
- [7] Hutchinson, Ian, *James Clerk Maxwell and the Christian Proposition*, MIT IAP Seminar: The Faith of Great Scientists (Jan 1998, 2006), available on the internet at <http://silas.psfc.mit.edu/maxwell/maxwell.html>.
- [8] Theerman, Paul, *James Clerk Maxwell and Religion*, Am. J. Phys., **54**(4) (April 1986).
- [9] McNatt, Jerrold, *James Clerk Maxwell's Refusal to Join the Victoria Institute*, Perspectives on Science and Christian Faith : journal of the American Scientific Affiliation, Vol.56, no.3 (September 2004), pp.204–215, available on the internet at <http://www.asa3.org/asa/pscf/2004/PSCF9-04McNatt.pdf>.
- [10] *James Clerk Maxwell*, Wikipedia article, http://en.wikipedia.org/wiki/James_Clerk_Maxwell.