Blaise Pascal

His Life and His Faith

George W. Benthien

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Email: georgewb@gbenthien.net

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Blaise Pascal (1623–1662)

Introduction

Who was Blaise Pascal? Was he a mathematician, an inventor, a scientist, an engineer, a literary giant, a philosopher, a theologian? The answer to all these is yes. In his short lifetime (39 years) he made important contributions in all these areas.

In mathematics his biggest contribution was probably his collaboration with Pierre de Fermat to form the beginnings of probability theory. Most students of mathematics are familiar with an arrangement of numbers called Pascal's triangle. Pascal didn't actually invent this triangle, but he made extensive use of it in the combinatorial problems associated with probability and was the first to publish a scientific paper dedicated solely to this triangle and its properties. In addition, he also derived a theorem that is an important part of projective geometry and made important contributions to finding the area and centroid of segments bounded by a famous curve called a cycloid.

His biggest invention was probably the first mechanical calculator called the Pascaline. This invention also involved a great deal of engineering skill in order to actually bring it into production. He is also credited with the invention of the syringe, the hydraulic lift, and the design of the first mass-transit system in Paris.

As a scientist he performed important experiments that helped to establish the existence of a vacuum. Although the existence of a vacuum is readily accepted today, it was hotly debated at the time. His experiments also established that atmospheric pressure decreases with altitude. He also established a law involving fluid pressure that is fittingly called Pascal's Law and is foundational to the field of hydraulics.

Pascal was a dedicated Christian and a member of a sect of Catholicism called Jansenism. This sect was a bitter rival of the Jesuits. Pascal composed a series of 18 letters that is collectively called Les Lettres Provinciales (The Provincial Letters) that defended Jansenism and made fun of

the Jesuits. This collection of letters is considered a masterpiece of French literature. Voltaire, who was a vocal opponent of Christianity, kept a copy by his bed and referred to it often. At his death Pascal was in the process of writing a defense of Christianity that was not completed. The collection of his notes have been published under the title Pensées (Thoughts). Pensées is well regarded as both a work in philosophy and in Christian apologetics. I'm sure many of you have seen quotations from Pensées such as

> *All of humanity's problems stem from man's inability to sit quietly in a room alone.*

> *The heart has its reasons of which reason knows nothing*

> *If we submit everything to reason our religion will be left with nothing mysterious or supernatural. If we offend the principles of reason our religion will be absurd and ridiculous … There are two equally dangerous extremes: to exclude reason, to admit nothing but reason.*

As you can see Blaise Pascal was a man of many talents. In the sections that follow we will present a brief biography of his life and look at his many accomplishments and his faith in more detail. In preparing this paper I looked at a great number of references, many of which are available online. Those that I found most helpful are listed in the Reference section at the end.

Brief Biography

Blaise Pascal was born on 19 June, 1623 in Clermont France. Clermont is located in central France and is surrounded by a chain of volcanoes. A picture of Clermont is shown in Figure 1.

Figure 1: Clermont: Pascal's Birthplace

France at this time was a major power in Europe and was a Catholic stronghold. France was in the process of rebuilding after several decades of war between French Catholics and Protestants.

Pascal's Father was Étienne and his mother was Antoinette. He had two sisters — Gilberte who was older and Jacqueline who was younger. Blaise became very sick when he was about two years old and nearly died. He suffered from ill health throughout his life. His mother died when he was about three years old and he was raised by his father. His father was a tax commissioner in Clermont. In addition to being a civil servant, Étienne was fluent in latin and greek and was an accomplished mathematician. In 1631 Étienne sold his government position in Clermont (a common practice in that day) and moved his family to Paris. He invested in government bonds to support his family.

Étienne had definite ideas on how to educate his children so they were home-schooled by him. He wanted them to be well grounded in languages, history, and philosophy before they took on more advanced subjects such as mathematics. He told Blaise that he couldn't study mathematics until he was 15 years old. However, Blaise was intrigued by geometry and secretly developed some of Euclid's theorems using his own terminology. When his father found out, he realized that his son was very gifted in this area and gave him a copy of Euclid's Elements to study.

Étienne was also a member of a small discussion group who were interested in the newly emerging field of natural philosophy (science). This group met in the quarters of a catholic priest Marin Mersenne, himself an accomplished mathematician. Members of this group included, Pierre de Fermat, Gilles Roberval, and Girard Desargues. They were also in correspondence with René Descartes, Christiaan Huygens, and Thomas Hobbes. The group discussed current topics in natural philosophy and mathematics including their own research. Blaise accompanied his father to the meetings when he was about 13 and began taking part in the discussions. The group could tell that this was a very gifted child.

In 1638 Cardinal Richelieu, the First Minister, entered France into the thirty years war. This was a power struggle between various catholic and protestant states. In order to help raise money for this war he defaulted on the government bonds. This was a financial disaster for Pascal's father Étienne. He joined with others in a protest rally. Cardinal Richelieu was enraged and jailed most of the protesters. Étienne escaped, but had to go into hiding. Jacqueline, Blaise's younger sister, was also a child prodigy in poetry and drama. She appeared in a performance before Cardinal Richelieu that moved him deeply. After the performance Jacqueline approached the Cardinal and begged him to forgive her father, and to allow him to return. The Cardinal not only forgave him, but offered him a high administrative position in Rouen. The family moved there with him.

Pascal's father faced a very difficult situation in Rouen. Cardinal Richelieu had imposed very high taxes to pay for the war and the people were angry. There were frequent riots. Étienne was a very capable and honest administrator and eventually gained the peoples trust. However, he was heavily burdened by the voluminous calculations involved in keeping up with the ever changing tax rates. Blaise would help his father with the calculations.

Blaise corresponded with the discussion group in Paris and was kept up-to-date on current events. Mersenne sent Blaise a copy of a book by Desargues on conic sections. Blaise was intrigued by the way Desargues used projections in this study. In 1642 Blaise sent Mersenne a copy of his first mathematical paper entitled ıEssai pour les coniques. In this paper Blaise derived a theorem on hexagons now known as Pascal's Theorem. This theorem states that if a hexagon (a six-sided polygon) is inscribed in a conic section (circle, ellipse, ...) and the opposite sides are extended until they intersect, then the three points of intersection will lie on a line. This theorem is pictured in Figure 2.
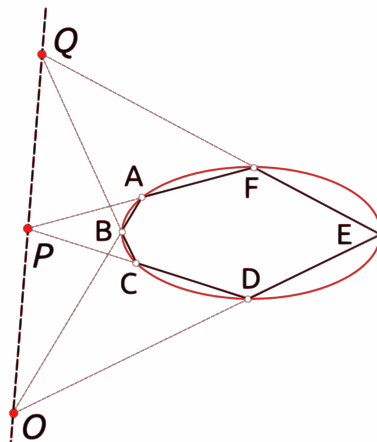

Figure 2: Pascal's Hexagon Theorem

Père Mersenne was overjoyed to see the first major result from this child prodigy. He passed the paper on to his many contacts throughout Europe. Pascal's theorem became an important result in the field of mathematics known as projective geometry.

Blaise was also able to help his father with the many tax calculations by inventing the first mechanical calculator known as the Pascaline. There is a picture of one in Figure 3.

Figure 3: The Pascaline

The design and operation of the Pascaline will be discussed in the section on Pascal's Contributions to Science and Engineering. Not only was the design of this device impressive, but it involved a great deal of engineering to bring it into production.

In 1642 Evangelista Torricelli performed an experiment that brought into question a long held belief in the scientific community. He took a glass tube that was closed at one end and filled it with mercury. He placed the tube inverted in a dish filled with mercury (see Figure 4).
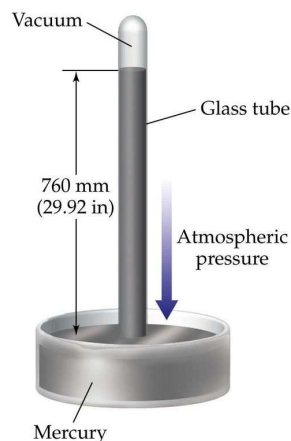

Figure 4: Torricelli Vacuum Experiment

The mercury level dropped leaving a column of mercury in the tube and an apparently empty space at the top of the tube. Torricelli claimed that the top space was actually a vacuum and that the pressure of the air on the mercury in the dish was balancing the weight of the mercury column. This caused quite a controversy since most scientists at this time were followers of Aristotle and believed that a vacuum was impossible. Aristotle had claimed that nature abhors a vacuum.

Blaise's father along with a friend duplicated the experiment in 1646. At first Blaise doubted that the empty space was in fact a vacuum, but on reading Torricelli's explanation he became convinced. Before long he would conduct some crucial experiments validating Torricelli's claim.

In the winter of 1646 Étienne slipped on the ice and broke his hip. Two local bonesetters stayed for a period of time with the Pascals and cared for Étienne. These young men were members of

the Jansenist sect within the Catholic church. The Jansenists followed closely the teachings of Augustine and thus had many similarities with the reformed Calvinists. Through many conversations with the two young men the Pascal family converted to Jansenism. Blaise. in particular was very excited about his new-found faith. However, there was one of their teachings that Blaise had difficulty in accepting. They encouraged their members to abandon all earthly pursuits including science. Blaise felt that this would be a waste of the gifts God had given him. You can read more about Pascal's faith in the section The Faith of Blaise Pascal.

In 1656–1647, while his father was recovering from his injury, Blaise performed a number of public demonstrations of Torricelli's experiment with various liquids. In 1657 he returned to Paris accompanied by his sister Jacqueline. She cared for Blaise during his frequent illnesses. In Paris Blaise resumed his vacuum experiments. His latter experiments concentrated on Torricelli's explanation that it was air pressure applied to the mercury in the dish that was balancing the mercury column. His most famous experiment was the one carried out on the mountain Puy de Dome that illustrated that air pressure decreases with altitude and thus that the mercury column drops in height. You can read more about these experiments in the section Pascal's Contributions to Science and Engineering. During this time Blaise met twice with René Descartes to discuss their differing views on the possibility of a vacuum.

In the spring of 1649, Due to civil unrest in Paris, Blaise along with his sister Jacqueline and father Étienne returned to Clermont and stayed with the family of Gilberte, Blaise's other sister. They returned to Paris in November 1650 as the riots had died down. Blaise's father Étienne died on September 24, 1651. For many years Jacqueline had wanted to become a nun at Port-Royal Abbey, a stronghold of Jansenism. However, Étienne had refused to let her go. After her father's death Jacqueline entered the Abbey. Blaise was now, for the first time, all alone. The next few years were a difficult time for him.

Blaise became friends with an aristocrat the duc de Roannez. They shared a common interest in the intellectual topics of the day and both had a desire for an authentic Christian spirituality. The duke introduced Blaise to a new circle of friends. One of these friends was the chevalier de Méré who posed two gambling problems to Blaise. These problems resulted in a series of letters between Blaise and Pierre de Fermat that formed the beginning of probability theory. You can read more about these problems in the section Contributions to Mathematics.

On the 23rd of November 1654 Blaise had a profound spiritual experience that changed his life. Blaise had always approached God through his mind using reason. But on this night he experienced the presence of God in a way that changed his heart. He never told anyone about this, but he wrote an account of this experience on a piece of paper and sewed it into the lining of his coat. The paper was discovered after his death when they were going through his clothes. This night is often referred to as his "Night of Fire" because of the references to fire in his account. There are more details in the section The Faith of Blaise Pascal. There were several stories that were later told about his "Night of Fire." One story claimed that he had nearly died in a carriage accident which caused him to reassess his life. Another story claimed that he had been deeply moved by a sermon. No one really knows what happened that night except for what he had written Although he never told anyone about this spiritual experience, his friends could see a difference in his life. His anxiety and confusion were replaced by a calmness and a peace. He later told Jacqueline that he was now

ready to completely embrace Jansenism. However, he still continued to work in mathematics and science and remained friends with the duke and his entourage. His continuation of these 'worldly' pursuits caused many of the Jansenists to question his commitment.

The Jansenists were continually under attack in the Catholic church, primarily by the Jesuits. These attacks were now increasing in intensity. A committee appointed by the Pope and the faculty of the Sorbonne condemned five of the core Jansenist beliefs as heretical. Antoine Arnauld, the current leader of the Jansenists, tried to defend their position, but was having little success. In desperation they turned to Pascal who penned the Provincial Letters. There were 18 letters in this collection written over a two year period 1656–1657. Pascal didn't use theological arguments, but instead used comedy and sarcasm to mock the Jesuits and to engender sympathy for the Jansenists. The Provincial Letters were very popular and widely read. It is said that Voltaire, an opponent of Christianity, carried a copy with him and kept one at his bedside to serve as a reminder of what good writing looks like. You can read more about these letters in the section Contributions to Literature.

Pascal's health continued to worsen in 1658. As a distraction he worked on the famous mathematical curve the cycloid. He was able, among other things, to find the area and center-of-gravity of this curve. Pacal's contributions can be found in the section Contributions to Mathematics. Pascal also announced his plans to publish a defense of Christianity. He didn't believe that the common man could follow the usual theological arguments. Instead he wanted to present arguments in the language of the people. He was never able to finish this work, but his notes were collected and published after his death as the Pensés. There is more on the Pensées in the section Contributions to Literature.

Jacqueline Pascal died in October of 1661 shortly after being forced to sign a document drafted by the king condemning Jansenist beliefs. In 1662 Blaise instituted the first omnibus service in Paris. You can read more about this achievement in the section Contributions to Science and Engineering. Later that year he became seriously ill and died on August 19, 1662. He was buried inside the church Saint-Étienne-du-Mont and the family placed a small plaque on the wall near his tomb. A picture of the church where he is buried is shown in Figure 5 below.

Figure 5: Pascal Burial Site

Contributions to Science and Engineering

Pascal made a number of contributions to science and engineering. In this section we will describe some of the more importans ones.

Fluids and Hydraulics   Pascal performed a number of experiments dealing with fluids and hydraulics. He determined that the hydrostatic pressure in a fluid depended on the height of the fluid above it and not on the weight of the fluid. To illustrate this he put a very long small diameter tube in a barrel and filled the barrel with water. From a high position he poured water into the narrow tube. When the water in the tube reached a sufficient height the barrel burst from the pressure.
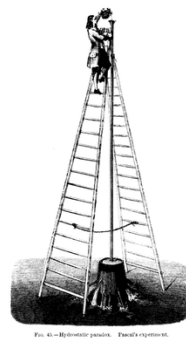

Figure 6: Pascal's Barrel Experiment

Pascal also formulated Pascal's Law that states

In a fluid at rest in a closed container, a pressure change in one part is transmitted without loss to every portion of the fluid and to the walls of the container.

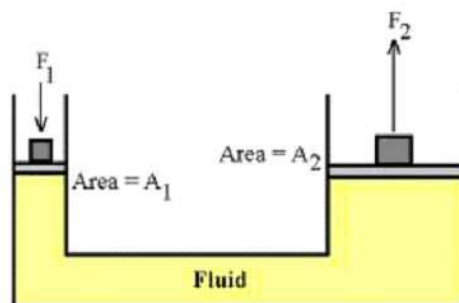This law is the basis of the hydraulic lift pictured in Figure 7.

Figure 7: Hydraulic Lift

Here a smaller weight produces a pressure change of F_1/A_1 in the fluid. This produces a force of $(F_1/A_1)A_2$ on the second piston. Thus the force is multiplied by A_2/A_1.

He also developed a simple syringe to illustrate his law. A plunger in a cylindrical tube was attached to a sphere that had multiple holes. Both the tube and the sphere were filled with water. This syringe is pictured in Figure 8 below

Figure 8: Pascal's Syringe

Applying pressure to the plunger cause water to squirt out of the holes in the attached sphere equally in all directions. This is an illustration of Pascal's law.

Computing   In order to help his father with his tax calculations Pascal designed and developed a mechanical calculator called a Pascaline. Pascal not only designed the device, but also supervised its production. There were about 50 of these devices produced, but only about 8–10 have survived to this day. A picture of a Pascaline is shown in Figure 9.

Figure 9: The Pascaline

The device was never a financial success. It was cheaper to hire workers to do the computations manually. A link to a You Tube video describing the operation of a Pascaline can be viewed at https://www.youtube.com/watch?v=3h71HAJWnVU.

Vacuum Experiments   Although it seems strange to us today, most scientists at the time of Pascal agreed with Aristotle that "Nature abhors a vacuum." In 1644 Evangelista Torricelli performed an experiment that challenged this belief. He took a glass tube that was closed at one end and filled it with mercury. Covering the open end he immersed it in a dish of mercury. When the cover was removed the mercury fell until there was a column about 30 inches high remaining in the tube. Figure 10 shows this experiment.

The question was "what is the empty space above the mercury column?" Torrecelli believed it was a vacuum. His explanation was that the air exerted a pressure on the mercury in the dish
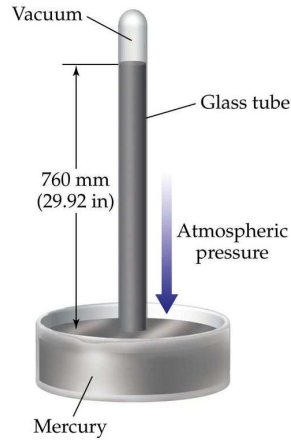
Figure 10: Torricelli Vacuum Experiment

that was balanced by the weight of the column of mercury. His explanation was not generally accepted. Many scientists thought there was some invisible substance in this apparently empty space. Possibly some type of vapor had leaked in.

Pascal heard about the experiment and duplicated it in 1646. Pascal duplicated the experiment with water and with wine. Since these substances weighed much less than mercury he had to construct a long glass tube about 45 feet high. The followers of Aristotle thought that wine was more vaporous than water and therefore the vapor in the void above the column should push the wine down farther than the water. However, wine is lighter than water and actually left a smaller void than water.

Pascal performed another interesting experiment. The apparatus is shown in Figure 11 below. The apparatus was filled with mercury and inverted as in the other experiments. A hole near the top was initially covered. The mercury in the lower straight section formed a column as in the other experiments. However, there was some mercury left in the curved trap. When the hole was uncovered the mercury column went down, but the mercury in the trap formed a column in the upper tube. The mercury in the lower tube went down since there is now air pressure on both side of the mercury column so there is nothing to balance the weight of the mercury column. The mercury in the upper column went up since there is now nothing to balance the air pressure until the mercury column is formed.
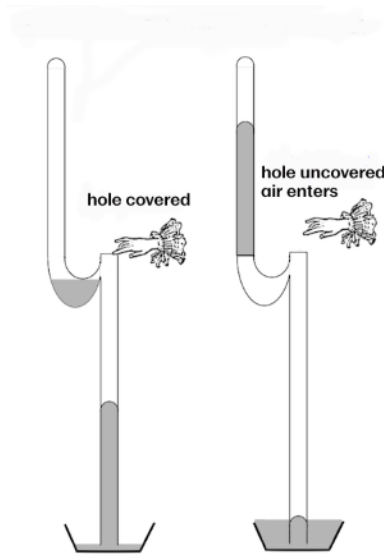
Figure 11: Another Vacuum Experiment

Pascal's most famous vacuum experiment involved taking a Torricelli barometer up to the top of a 3000 ft. mountain to see how the height of the mercury column varied with altitude. Pascal reasoned that the air pressure was due to the weight of the air above us. Therefore, it was reasonable to assume that the air pressure would be less at a high altitude. On September 19, 1648 Pascal had his brother-in-law and several trusted friends take measurements at the bottom of the mountain Puy de Dome near Clermont and then take the device to the top recording measurements as they went.

Figure 12: Puy de Dome Experiment

They found that the height of the mercury column was 711 mm at the bottom of the mountain and 627 mm at the top, a difference of over 3 inches. This was added proof that it was air pressure acting on the surface of the mercury reservoir that was balancing the weight of the mercury column.

Thus, Torricelli's barometer could be used to measure air pressure. A numerical value for the air pressure could be obtained by dividing the weight due to the column of mercury by the cross-sectional area of the tube. Pascal then multiplied this pressure by the surface area of the earth to obtain an estimate of the total weight of the atmosphere. He was within 30% of the generally accepted value today.

Pascal Initiates Omnibus System   In 1662 Pascal had the idea of creating an omnibus system in Paris. He secured financing from some of his friends and the king granted him monopoly status. The system started with seven horse-drawn vehicles running along regular routes. Each coach was capable of carrying six to eight passengers. An artist's rendition is shown in Figure 13 below.

Figure 13: A Pascal Omnibus Carriage

The omnibus system was a big success at first, but soon the novelty wore off. At this time only the nobility and gentry were allowed to ride the coaches. Soldiers and peasants were not allowed. As the aristocrats were not dependent on the system, the business was not sustainable. By 1675 the omnibus system was out of business. This proved to be an idea that was ahead of its time.

Pascal and Mathematics

In this section we will look at several areas in mathematics where Pascal made an important contribution

Geometry   When Pascal was 16 years old he wrote a paper entitled *Essay pour les coniques*. In it he presented Pascal's Theorem that states that the intersections of opposite sides of a hexagon inscribed in a conic section are colinear. This is a fundamental theorem in what is now called projective geometry.
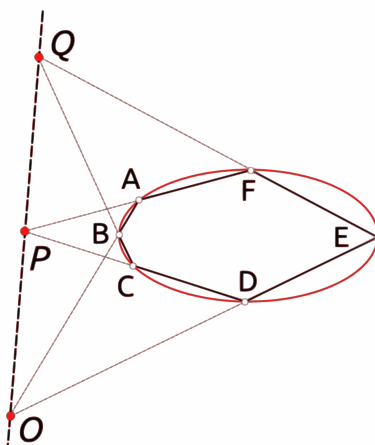

Figure 14: Pascal's Hexagon Theorem

Probability Theory   It is generally accepted that probability theory had its origin in a series of letters between Blaise Pascal and Pierre de Fermat. The subject of these letters was several gambling questions posed to Pascal by Antoine Gombaud, chevalier de Méré. The chevalier de Méré had been wagering that he could roll a six with a single die in four rolls. He had won for a while, but was now losing. He switched to a different game where he bet that he could roll a double six with two dice in 24 rolls. He was losing more than he was winning. He asked Pascal if he was just unlucky or was he making a stupid bet. Pascal showed him that he had a slight advantage in the first game, but was at a slight disadvantage in the second game. In the first game the probability of not rolling a six on a single roll was 5/6. Therefore, the probability of not rolling a six in four rolls was $(5/6)^4 = 0.4823$. It follows that the probability of rolling at least one six in four rolls is $1 - (5/6)^4 = 0.5177$ — a slight advantage. Similarly, in the second game the probability of rolling a double six in 24 rolls is $1 - (35/36)^{24} = 0.49144$ — a slight disadvantage.

The next question posed to Pascal was more challenging and was the subject of several letters between Pascal and Fermat. This question involved the fair distribution of the pot in an unfinished game. The game involved setting a number of points for a win. The players rolled the dice and the highest roll received a point. This was repeated until one of the players had the agreed upon

number of points. However, what should be done if the game was discontinued before one of the players won? How should the pot be divided? A discussion of the various solutions offered by Pascal and Fermat is contained in Appendix B: Analysis of Unfinished Game Problem.

In solving the combinatorial problems that arise in probability calculations Pascal made great use of the triangular array of numbers now known as Pascal's triangle. A discussion of this triangle and its properties is contained in Appendix A: Pascal's Triangle.

The Cycloid   A cycloid (in French la roulette) is the curve generated by a point on a circle as the circle rolls along a line (see Figure 15).
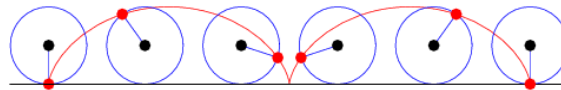
Figure 15: The Cycloid Curve

This curve had been studied by many of the great minds of the time including Galileo, Descartes, and Roberval. Pascal found a way to find the area and center of gravity of any segment under the curve. He also showed how to calculate the volume and surface area of the solid obtained by rotating the cycloid about the line it was rolling on. This work was a predecessor of the more general problem in calculus of finding the area under a curve by integration.

Contributions to literature

Pascal was not only an outstanding thinker, but he was also an outstanding writer. He wanted his ideas to be understood by an ordinary person. In this section we describe his two most famous compositions.

Lettres Provinciales   The Provincial Letters (Les Lettres Provinciales) is a collection of 18 letters that were written by Pascal in 1656–1657. This collection is considered a classic of French Literature. It is said that Voltaire, certainly not a friend of Christianity, kept a copy by his bed as an example of what good writing looks like.

The situation that led to the writing of these letters was the condemnation of Antoine Arnauld, the leader of the Jansenist sect within Catholicism, by both the Sorbonne and a special committee set up by the Pope. It was held that five of the Jansenist beliefs advocated by Arnauld were heretical. The Jansenists followed the teachings of Augustine that God alone determined who would be saved and that God's grace offered to his elect could not be refused, but would accomplish its desired effect without any help from the individual. The two most powerful groups in the Catholic church at this time were the Jesuits and the Dominicans. The Jesuits believed that God's grace was offered to all, but each individual had the freedom to accept or reject it. This grace was called God's Sufficient Grace. The Dominican's accepted the idea of God's sufficient grace offered to all, but that it required God's Efficacious Grace, offered only to the elect, to take advantage of it. Pascal used a comedic style and sarcasm to present his arguments. His arguments were directed most strongly against the Jesuits who were the most outspoken opponents of Jansenism.

The first four letters are primarily a defence of the views of Arnauld. Letters 5–10 are directed toward the moral laxity of the Jesuits and their justification of it. The remaining letters are a direct attack against the Jesuits where Pascal abandons the comedic style of the earlier letters and writes with great passion. The first 10 letters are presented as a conversation between an ordinary resident of Paris and a friend in the provinces concerning the events involving the Sorbonne and the religious leaders that are in the news. The later letters are addressed directly to the Jesuit Fathers. Below is an excerpt from the second letter where the speaker is addressing a Dominican Father.

> But, to return to the point, father; this grace given to all men is sufficient, is it not?
> *Yes*, said he.
> And yet it has no effect without efficacious grace?
> *None whatever*, he replied.
> *And all men have the sufficient,* continued I, *and all have not the efficacious?*
> *Exactly,* said he.
> *That is,* returned I, *all have enough of grace, and all have not.*
> …
> this grace suffices, though it does not suffice—that is, it is sufficient in name, and insufficient in effect! In good sooth, father, this is particularly subtle doctrine! Have you forgotten, since you retired to the cloister, the meaning attached, in the world you have quitted, to the word sufficient ?

The Provincial Letters were very popular with the general public, even after they were eventually banned by the Pope. This was due in part to French nationalism and the close ties of the Jesuits with the Pope who didn't reside in France.

Pensées   At his death Pascal was in the process of writing a defense of Christianity that was not completed. However, many of the notes he had prepared have been recovered. We do not know the order he planned to present the material and we don't know if all of the notes would have made into the final publication. The notes have been collected together in a volume called Pensées (thoughts).

The material in the Pensées has been used both by philosophers and by Christian apologists. Probably the most famous selection from the Pensées is known as Pascal's wager. Below is a sample of Pascal's argument.

> Let us then examine this point, and let us say: "Either God is or he is not." But to which view shall we be inclined? Reason cannot decide this question. Infinite chaos separates us. At the far end of this infinite distance a coin is being spun which will come down heads or tails. How will you wager? Reason cannot make you choose either, reason cannot prove either wrong.
> …
> Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist.

Pascal's argument was not designed for those seriously seeking to know God, but was designed for the secular population who avoided thinking about God and other ultimate topics through diversions such as gambling. It's goal was simply to help them see that, even from a secular point of view, belief in God was not unreasonable. It was not a method for evangelism, but could be considered as pre-evangelism. Pascal wanted to get them thinking about God's existence.

The Faith of Blaise Pascal

Blaise Pascal was a committed Christian who took his faith seriously. As mentioned earlier he belonged to the Jansenist sect of the Catholic Church and his beliefs were greatly influenced by his association with this group. Let's look at some of the Jansenist beliefs. As far as salvation was concerned the Jansenists followed a very literal interpretation of Augustine and thus had a lot in common with the Calvinists among the protestant reformers. For example, they believed in the following

1. all share in Adam's original sin

2. man is totally depraved and is incapable of seeking or desiring God on his own

3. salvation was only possible for the elect chosen by God

4. Christ's grace offered to the elect could not be resisted

5. Christ's atoning death only applied to the elect

Here is a quote by Pascal on the above beliefs

> …*men are saved or damned according as to whether it has pleased God to choose them as recipients of this [efficacious] grace from out of the corrupt mass of men, in which He could with justice abandon them all.*

The Jansenists differed from Calvinists in that they believed that the salvation of the elect was not secure and could be lost. They also accepted most of the other beliefs of the Catholic Church not related to salvation. Beliefs such as apostolic succession, sainthood, veneration of Mary, and the nature of the sacrament of communion.

Unlike the Calvinists, the Jansenists taught a form of piety that involved withdrawal from the world and its diversions. This was the one belief of the Jansenists that Pascal had great difficulty with. He couldn't see why it was necessary to abandon mathematics and science when his abilities in these fields were a gift from God. The fact that he remained active in these fields caused many Jansenists to question his commitment.

Throughout most of his life Blaise approached religion the same way he approached everything else — through reason. On the night of November 23, 1654 this all changed. Blaise experienced a mystical encounter with God known as his "night of fire." He didn't tell anyone what happened that night, but he wrote down an account on a piece of paper and sewed it into the lining of his jacket. The paper was found after his death as they were going through his clothes. The details of what happened leading up to this encounter are not known, but here is what he wrote.

The year of grace 1654,

Monday, 23 November, feast of St. Clement, pope and martyr, and others in the marty-rology. Vigil of St. Chrysogonus, martyr, and others. From about half past ten at night until about half past midnight,

FIRE.

GOD of Abraham, GOD of Isaac, GOD of Jacob
not of the philosophers and of the learned.
Certitude. Certitude. Feeling. Joy. Peace.
GOD of Jesus Christ.
My God and your God.
Your GOD will be my God.
Forgetfulness of the world and of everything, except GOD.
He is only found by the ways taught in the Gospel.
Grandeur of the human soul.
Righteous Father, the world has not known you, but I have known you.
Joy, joy, joy, tears of joy.
I have departed from him:
They have forsaken me, the fount of living water.
My God, will you leave me?
Let me not be separated from him forever.
This is eternal life, that they know you, the one true God, and the one that you sent,
Jesus Christ.
Jesus Christ.
Jesus Christ.
I left him; I fled him, renounced, crucified.
Let me never be separated from him.
He is only kept securely by the ways taught in the Gospel:
Renunciation, total and sweet.
Complete submission to Jesus Christ and to my director.
Eternally in joy for a day's exercise on the earth.
May I not forget your words. Amen.

On this night God touched Blaise's heart and he was not the same afterwards. His friends and family could see that he had changed, but they didn't know the cause of this change. Blaise committed himself all the more to the Jansenist cause, but he never completely gave up his non-Christian friends and his involvement in mathematics and science.

One of the evidences of his changed life was an increased compassion for the poor. Even though his physical health was declining, he often got out of bed and walked among the poorest of the poor in Paris and gave them what money he had. His sister Gilberte told the following story:

One day when he was coming home from Mass he encountered a fifteen year-old girl begging for money. She told him that her father had died and her mother was near death. Blaise took her to the rectory and gave the priest money to take care of her. He begged the priest to find a safe place for the girl to live. In a few days he sent a woman servant to check on the girl and to provide more money. Blaise never told the priest or the girl his name, so the girl never knew who it was that had been so kind to her.

Towards the end of his life Blaise was in the process of writing a defense of Christianity. This was never finished, but his notes were collected and published as the Pensées. His two main purposes in these writings was to show the wretchedness of man without God and the happiness of the life with God. There is more about the Pensées in the section Contributions to Literature. The Pensées are often quoted by Christians of all denominations as well as by philosophers.

Throughout his life Blaise Pascal loved mathematics and science, but he grew to love God even more.

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Appendix A: Pascal's Triangle

Most students of mathematics are familiar with a triangular array of numbers called Pascal's triangle. The form usually found in textbooks is shown in Figure 16.

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
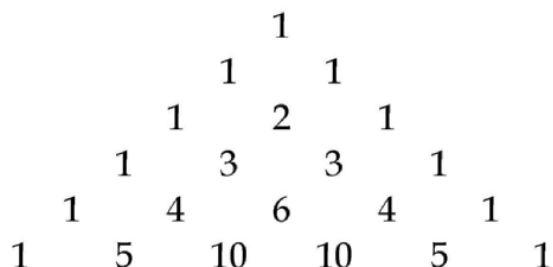1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Figure 16: Pascal's Triangle

Pascal wrote a paper dealing with this triangle and its properties entitled *Traité Du Triangle Arithmétique*. In this paper I will use a different arrangement of this triangle as shown in Figure 17.

$$
\begin{array}{cccccc}
1 & & & & & \\
1 & 1 & & & & \\
1 & 2 & 1 & & & \\
1 & 3 & 3 & 1 & & \\
1 & 4 & 6 & 4 & 1 & \\
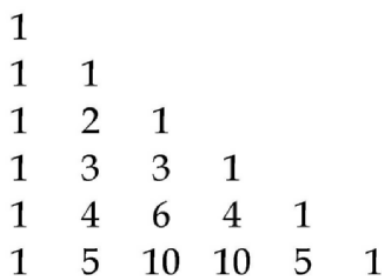1 & 5 & 10 & 10 & 5 & 1
\end{array}
$$

Figure 17: Pascal's Triangle (alternate form)

This form lends itself better to row and column notation. We will number the rows and columns starting with 0. Let (n, r) denote the element in the n-the row and the r-the column. The numbers interior to the triangle satisfy the relation

$$(n, r) = (n - 1, r) + (n - 1, r - 1), \tag{1}$$

i.e, each interior element is obtained by adding the element one row above it in the same column to the element that is one row above and one column to the left. In addition, $(n, 0) = (n, n) = 1$ for all n and $(n, r) = 0$ for $r > n$.

We will show that the elements of this triangle are equal to the binomial coefficients. The binomial coefficients are the coefficients $\binom{n}{r}$ in the expansion

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r. \tag{2}$$

Clearly

$$(1+x)^n = \overbrace{(1+x)(1+x)\cdots(1+x)}^{n \text{ factors}}$$

The term x^r is obtained by taking x from r of the factors and 1 from the remaining $n - r$ factors. Therefore, the coefficient $\binom{n}{r}$ must be the number of different ways of choosing r x's from the n factors. Since there is nothing special about the x's, $\binom{n}{r}$ must be equal to the number of combinations of n objects taken r at a time.

In order to show that $(n, r) = \binom{n}{r}$ we must show that the binomial coefficients satisfy

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1} \tag{3}$$

with $\binom{n}{0} = \binom{n}{n} = 1$ for all n and $\binom{n}{r} = 0$ for $r > n$. This is the same relation that the elements (n, r) satisfy [see Equation 1].

To derive this recursion relation recall that $\binom{n}{r}$ was the number of combinations of n objects taken r at a time. If we want to find the number of groups of r objects taken from a group of n objects, we can break the process into two parts. We divide the n objects into a group A of $n - 1$ objects and a group B with one object. If we find all the groups of r objects taken from group A, these groups of r objects are part of the ones considered in $\binom{n}{r}$. There are $\binom{n-1}{r}$ of these groups. What we are missing are those containing the object in group B. Each of those involves $r - 1$ objects from group A. Clearly there are $\binom{n-1}{r-1}$ of these. This argument establishes the above recursion relation. In addition, it can be seen from the binomial expansion that $\binom{n}{0} = \binom{n}{n} = 1$ for all n. Moreover, $\binom{n}{r} = 0$ for $r > n$ since there are no powers of x larger than n.

Since (n, r) and $\binom{n}{r}$ satisfy the same generating equations, we must have $(n, r) = \binom{n}{r}$. Pascal used the binomial coefficients and the associated Pascal triangle to solve many of the combinatorial problems involved in probability calculations.

The binomial coefficients have a number of other properties. For example, if we let $x = 1$ in Equation 2, then we get

$$\sum_{r=0}^{n} \binom{n}{r} = 2^n, \tag{4}$$

i.e, the sum of the elements in the n-the row of Pascal's triangle is 2^n. There is also the following

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direct expression for calculating the binomial coefficients

$$\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{r!}.$$

(5)

To see this we will start with ordered arrangements (permutations) and then proceed to unordered combinations. To find all the ordered arrangements of n elements, we have n choices for the first entry, $n-1$ choices for the second , down to 1 choice for the n-the entry. Thus, there are $n(n-1)\cdots 1 = n!$ ordered arrangements of n elements. Similarly, there are $n(n-1)\cdots(n-r+1)$ ordered arrangements of length r taken from n elements. However, in the binomial coefficients we don't want ordered arrangements, but unordered sets. For each unordered collection of r elements taken from n elements there are $r!$ arrangements of these r elements. Thus, the number of unordered sets of r elements taken from n elements can be obtained by dividing the number of ordered arrangements of r elements taken from n elements by $r!$. This completes the proof of Equation 5.

From Equation 5 we can easily derive several other recursion relations, e.g.,

$$(r+1)\binom{n}{r+1} = (n-r)\binom{n}{r}$$

(6)

and

$$r\binom{n}{r} = n\binom{n-1}{r-1}.$$

(7)

Pascal derived these relations along with a number of others in his paper.

Appendix B: Analysis of Unfinished Game Problem

The unfinished game problem (sometimes called the problem of points) grew out of a gambling question posed to Blaise Pascal. It involved a game between two players having the following rules

1. Both players put the same amount into a pot that will go to the winner.

2. The players agree on the number of points that will win the game.

3. To win a point each player roles the dice and the one with the higher total wins the point. In case of a tie the players roll again.

4. This continues until one of the players reaches the agreed upon number of points. This player then wins the pot.

The following question was presented to Blaise Pascal:

> *Suppose the game was interrupted before either player reached the point goal. How should the pot be fairly divided among the two players?*

Pascal discussed this problem with another mathematician Pierre de Fermat in a series of letters and in the process formed the beginnings of probability theory. The two came up with different approaches to the question that led to the same answer. They both agreed that the past history was not important. All that mattered was the number of points each needed to win. Suppose that the first player needed m points to win and the second player needed n points to win. Both Pascal and Fermat agreed that one of the players would be guaranteed to win in $n + m - 1$ additional rounds. To see this we must find the path to a win involving the most rounds. The longest path must eventually lead to the situation where both players need just 1 point to win. For one player must eventually reach the point where he needs 1 point to win. If the other player is not there yet, then there can be more rounds until that player needs 1 point to win. It requires m-1 wins from one player and n-1 wins from the other player or $m + n - 2$ rounds to reach the place where both players need 1 point to win. It follows that one of the players will always win in $m + n - 1$ rounds.

Fermat's Approach   Fermat's approach was to enumerate the various possibilities for $n + m - 1$ additional rounds and then determine the number that represented a win for each player. Consider the case $m = 2$ and $n = 3$. Then one of them will surely win in $2 + 3 - 1 = 4$ rounds. If F represents a round win for the first player and S represents a round win for the second player, then the 16 possibilities for the four rounds are

$$\begin{array}{cccccccc}
\text{FFFF} & \text{FFFS} & \text{FFSF} & \text{FSFF} & \text{SFFF} & \text{FFSS} & \text{FSFS} & \text{SFFS} \\
\text{FSSF} & \text{SFSF} & \text{SSFF} & \text{FSSS} & \text{SFSS} & \text{SSFS} & \text{SSSF} & \text{SSSS}
\end{array}$$

The first player wins the pot in 11 of the 16 cases and the second player wins the pot in 5 of the 16 cases. Thus, it seems fair to give the first player 11/16 of the pot and the second player 5/16 of the pot. Of course the enumeration method of Fermat could be very tedious if m and n are large.

Some objected to extending the game to $n + m - 1$ additional rounds when one of the players may win before that. Even though that is true, the full extension is convenient in order to guarantee that each of the outcomes has the same probability. In addition it was argued that the extension would not change the probability of either player winning. This can be seen for the case we just considered in the table below. This table lists the possible ways the first player could win along with the probability of each outcome (the probability of winning or losing a round is $\frac{1}{2}$).

Results	Probability
FF	1/4
FSF	1/8
SFF	1/8
FSSF	1/16
SFSF	1/16
SSFF	1/16

The sum of these probabilities is 11/16 which is what we obtained before. Later we will see a different approach by Pascal using recursion in which the logic behind the conclusion is clearer.

Pascal's Refinement of Fermat's Approach Pascal came up with an alternate way to look at the enumeration method of Fermat using binomial coefficients and a triangle of numbers usually called Pascal's triangle (see appendix A). The number of rounds that contain two F's is given by $\binom{4}{2}$. To see this we can describe each pair of F's by describing their position in a set of four letters. Let $\{1, 3\}$ represents an F in the first and in the third position. We do not distinguish between $\{1, 3\}$ and $\{3, 1\}$. Clearly, the number of all such distinct positions is the number of combinations of the numbers 1, 2, 3, and 4 taken two at a time without respect to order or $\binom{4}{2}$. Similarly, $\binom{4}{3}$ is the number of rounds containing three F's. It follows that the number of rounds containing at least two F's is given by $\binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 6 + 4 + 1 = 11$. This is the number we obtained previously by enumeration and counting.

Pascal's Recursion Method Pascal came up with another method using recursion. Let $e(m, n)$ represent the share due the first player when he has m points to a victory and the other player has n points to a victory. Consider one more hypothetical round. After that round the share of the first player is either $e(m - 1, n)$ or $e(m, n - 1)$. Since both outcomes are equally likely, it seems fair to set

$$e(m, n) = \tfrac{1}{2} e(m - 1, n) + \tfrac{1}{2} e(m, n - 1).$$

Consider the case where $m = 2$ and $n = 3$. We then have

$$e(2,3) = \tfrac{1}{2}e(1,3) + \tfrac{1}{2}e(2,2)$$
$$e(1,3) = \tfrac{1}{2}e(0,3) + \tfrac{1}{2}e(1,2)$$
$$e(1,2) = \tfrac{1}{2}e(0,2) + \tfrac{1}{2}e(1,1)$$

We start with the last equation and work our way back up to the first equation. Since $e(0,2) = 1$ and $e(1,1) = 1/2$, we have

$$e(1,2) = 1/2 + 1/4 = 3/4.$$

Since $e(0,3) = 1$, it follows that

$$e(1,3) = 1/2 + 3/8 = 7/8.$$

Since $e(2,2) = 1/2$, it now follows that

$$e(2,3) = 7/16 + 1/4 = 11/16.$$

This is the same answer that Fermat obtained. The quantity e used by Pascal is what we would now call expected value. This was a new and important concept introduced by Pascal.